The vehicle routing problem with simultaneous pick-up and deliveries and a GRASP-GA based solution heuristic

Arif Volkan Vural

THE VEHICLE ROUTING PROBLEM WITH SIMULTANEOUS

PICK-UP AND DELIVERIES AND A GRASP-GA

BASED SOLUTION HEURISTIC

By

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In this thesis, the vehicle routing problem and one of its variants, the vehicle routing problem with simultaneous pick up and deliveries (VRPSPD) are studied. The traditional vehicle routing problem (VRP) consists of constructing minimum cost routes for the vehicles to follow so that the set of customers are visited only once. A lot of effort has been devoted to research on developing fast and effective solution methods for many different versions of this problem by different majors of engineering profession. Thus, a structuring effort is needed to organize and document the vast literature so far has accumulated in this field.

Over its lifespan the VRP literature has become quite disjointed and disparate. Keeping track of its development has become difficult because its subject matter transcends several academic disciplines and professions that range from algorithm design to traffic management. Consequently, this dissertation begins with defining VRP's domain in its entirety, accomplishes an all-encompassing taxonomy for the VRP

literature, and delineates all of VRP's facets in a parsimonious and discriminating manner. Sample articles chosen for their disparity are classified to illustrate the descriptive power and parsimony of the taxonomy.

Next, a more detailed version of the original problem, the VRPSPD is examined and a more abstract taxonomy is proposed. Additionally, two other existing classification methodologies are used to distinguish all published VRPSPD papers on their respective research strategies and solution methods. By using well-organized methods this study provides a solid multidimensional identification of all VRPSPD studies' attributes thus synthesizing knowledge in the filed.

Finally, a hybrid meta-heuristic solution algorithm for the VRPSPD problem is presented. To solve this NP-hard vehicle routing problem a GRASP initiated hybrid genetic algorithm is developed. The algorithm is tested on two sets of benchmark problems from the literature with respect to computational efficiency and solution quality. The effect of starting with a better initial population for the genetic algorithm is further investigated by comparing the current results with previously generated ones. The experimental results indicate that the proposed algorithm produces relatively good quality solutions and a better initial population yields a reduction in processing cycles.

Key words: vehicle routing problem, genetic algorithms, GRASP, metaheuristic

DEDICATION

I would like to dedicate this research to my parents, Fatma and Can Vural, and to my sister Yeliz Vural Erim.

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CHAPTER I

INTRODUCTION

During the last decade, transportation and distribution systems became increasingly complex. This development is partly due to the enhanced capability of micro computers, whose speed have doubled once every couple of years. Another factor that complicates the distribution problems is the changing environment of today's businesses. As supply chain and just-in-time concepts become more prominent, transportation of goods in rather smaller batches, faster and more efficient than ever before is getting utmost importance. These facts have caused an increasing interest in vehicle routing problems, their variants, and fast methods to solve them with reasonably good solution quality.

The transportation of goods or humans from one location to another has been a major problem to solve during distribution network design. The well-known Vehicle Routing Problem (VRP) arises through this major design need to utilize the resources of a distribution network to its utmost capacity. There exist a huge number of qualitative as well as quantitative criteria to take care of during analyzing phase of the problem. However, due to modeling concerns, only a set of aspects may be studied in a single problem.

Since it was first formulated in 1959 by Dantzig and Ramser, the VRP has appealed much attention by the operations research academia. The VRP is a resembling problem to the "Traveling Salesman Problem" (TSP). In the TSP, the aim is to find the shortest trip for a salesman who is supposed to cover all customers starting at an initial location and obliged to return to some definite location, which is usually the initial location. The "VRP" is the name given to the class of problems that not only comprises the TSP but also adds some new features that add up to the current complexity of the TSP.

Over its lifespan the VRP literature has become quite disjointed and disparate. Keeping track of its development has become difficult because its subject matter transcends several academic disciplines and professions that range from algorithm design to traffic management. The traditional vehicle routing problem consists of constructing minimum cost routes for the vehicles to follow so that the set of customers are visited only once. A lot of effort has been devoted to research on developing fast and effective solution methods for many different versions of this problem by different majors of engineering profession. Thus, a structuring effort is needed to organize and document the vast literature so far has accumulated in this field. Consequently, this dissertation begins with defining VRP's domain in its entirety, accomplishes an all-encompassing taxonomy for the VRP literature, and delineates all of VRP's facets in a parsimonious and discriminating manner. Sample articles chosen for their disparity are classified to illustrate the descriptive power and parsimony of the taxonomy.

In the third and fourth chapters of this study, one special configuration of the VRP, known as the vehicle routing problem with simultaneous pick-up and deliveries (VRPSPD) is discussed. The problem comprises many customers or “nodes” to be served by a fleet of vehicles of homogeneous type and limited capacity. The vehicles deliver items to customers from the depot and pick-up loads to be delivered back to the depot at the end of the trip. The size of the picked up and delivered items are identical and they consume the same amount of capacity on each truck. Delivery and pick-up locations are unique and feeding a customer with anything picked up at a node other than the main depot is strictly avoided. The objective is to minimize the total distance covered by the fleet during service. Some instances of this type of problem may be observed in distribution networks of bottled spring water in re-collectable containers, industrial gas distribution-collection in refillable tanks, liquefied petroleum gas distribution in commercial containers from wholesalers to retailers, and so on.

A thorough examination and classification of the extant literature of the restricted domain of VRP with pick-up and deliveries is provided in the third chapter. A more focused taxonomy version for this field is proposed and the major problem area this dissertation focuses on, the VRPSPD is contained among similar problem types. Following this, two other existing classification methodologies from literature are used to distinguish all published VRPSPD papers on their respective research strategies and solution methods. By using well-organized methods, the third chapter provides a solid multidimensional identification of all VRPSPD studies’ attributes thus synthesizing knowledge in the filed.

Finally, in the fourth chapter, a hybrid meta-heuristic solution algorithm for the VRPSPD problem is presented. To solve this NP-hard vehicle routing problem a greedy randomized adaptive search procedure (GRASP) initiated hybrid genetic algorithm is developed. The algorithm is tested on two sets of benchmark problems from the literature with respect to computational efficiency and solution quality. The effect of starting with a better initial population for the genetic algorithm is further investigated by comparing the current results with those from the literature. The experimental results indicate that the proposed algorithm produces relatively good quality solutions and a better initial population yields a reduction in processing cycles.

CHAPTER II

THE VEHICLE ROUTING PROBLEM: A TAXONOMIC REVIEW

2.1. Introduction

The primary objective of this chapter is to present a taxonomic framework for defining and integrating the domain of the extant VRP literature in terms that are operationally meaningful. Such a framework will enable the classification of all VRP papers, which in turn will enable, among other things, systematic identification of voids in the literature and thus lead to potential topics for research. During the last three decades advances in computational capability, together with computer accessibility, have spawned major advances in algorithm development. These developments have enabled major advances in addressing the VRP. The need to improve vehicle routing was, in turn, spurred on by explosive developments in theory and applications of supply chain management. So, it should come as no surprise that the VRP literature has grown exponentially as a result of the above.

The literature of history and philosophy of science is replete with admonitions that, as important as it is to publish knowledge gained from good research in a given subject area, it is even more important to periodically reflect and assess where the field has been, where it is heading, and what, if anything, should be done to change that field's course. This indicates the need for some form of meta-research (Abbott 1988; Cooper 1984, 1988, 1989, 1998).

> In primary [social science] research, data are collected by asking people questions or observing their behaviour. In research synthesis, data are collected by conducting a search of reports describing past studies relevant to the topic of interest – (Cooper 1998).

Meta-research often serves other very important objectives as well. One of these is consolidating the knowledge in a given discipline; another is expanding the discipline's scope. Consolidation and expansion are not mutually exclusive; in fact, they are complimentary.

2.1.1. Vehicle Routing Problem Definition

The VRP can be represented as the following graph-theoretic problem. Let $G = (V, A)$ be a complete graph where $V = \{0, 1, \ldots, n\}$ is the vertex set and A is the arc set. Vertices $j = 1, \ldots, n$ correspond to the customers, each with a known non-negative demand, d_j, whereas vertex 0 corresponds to the depot. A non-negative cost, c_{ij}, is associated with each arc $(i, j) \in A$ and represents the cost of traveling from vertex i to vertex j. If the cost values satisfy $c_{ij} = c_{ji}$ for all $i, j \in V$, then the problem is said to be a symmetric VRP; otherwise, it is called an asymmetric VRP. In several practical cases the cost matrix satisfies the triangle inequality, such that $c_{ik} + c_{kj} \geq c_{ij}$ for any $i, j, k \in V$ (Toth and Vigo 1998).

The VRP consists of finding a collection of k simple circuits, each corresponding to a vehicle route with minimum cost, defined as the sum of the costs of the circuits' arcs such that:

i. each circuit visits vertex 0, i.e., the depot;

ii. each vertex $j \in V \setminus \{0\}$ is visited by exactly one circuit; and

iii. the sum of the vertices' demand visited by a circuit, does not exceed the vehicle capacity, C.

Figure 1.1 illustrates how a solution to a VRP would look like after routes are generated. The sketch shows the vertices to be served (customers), the edges (route segments), and the depot.

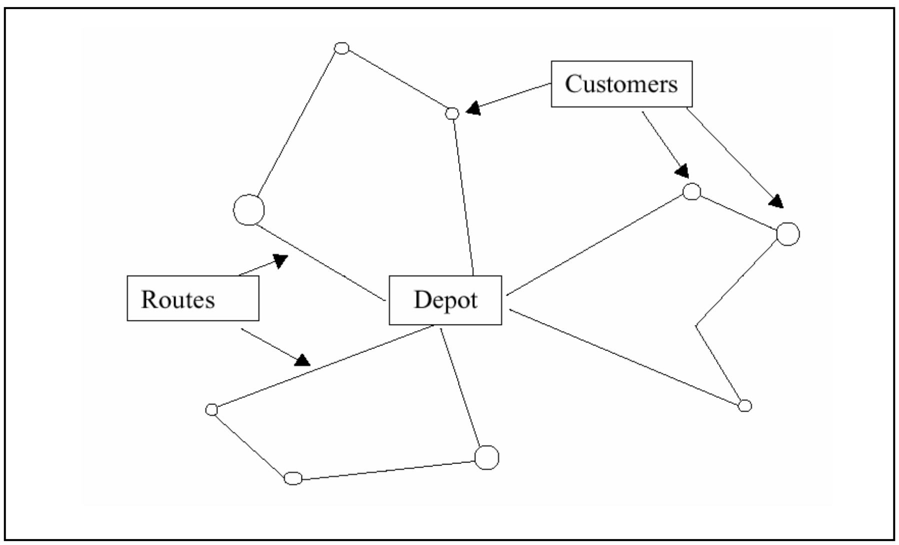

Figure 2.1 General representation of the Vehicle Routing Problem.

There exist quite many instances of VRP in our daily life. Partyka and Hall (2000) provide a non-exhaustive list of vehicle routing applications picked from the industry. Some of these are beverage delivery to bars and restaurants, currency delivery and scheduling at ATM machines, dynamic sourcing and transport of fuels, grease pickup from restaurants, home appliance repair service and delivery, internet-based home grocery delivery, milk pickup and inventory management , pickup of charitable donations from homes, portable toilet delivery, pickup and service, prisoner transportation between jails and courthouses, transport of urine samples from medical offices to laboratories, trash pickup and trans-shipments, wholesale distribution from warehouses to retailers, and postal delivery truck routing. Additionally, Psaraftis (1995) lists a non-exhaustive list of ten types of distribution problems including, but not limited to, the VRP and its dynamic version. Among them are: 1) delivery of petroleum products, industrial gases, and other products, 2) courier services, 3) tramp ship operations, 4) combined pick-up and delivery services, 5) dynamic routing of robots on shop floors, 6) para-transit (dial-a-ride) services route planning, and 7) share-a-cab services. Besides the problems listed above, one can find an enormous number of other distribution problems in the literature.

2.1.2. A Brief History of the VRP Literature

The paper by Dantzig et al. (1954), the first record in the VRP literature, studied a relatively large scale TSP and proposed a solution method. That study was followed by a great volume of other TSP papers. TSP can be shown to be a specific case of VRP. Clarke and Wright (1964) first incorporated more than one vehicle in the problem formulation. Consequently, this study may be considered as being first in the VRP literature as we know it.

The first paper bearing the phrase “vehicle routing” in its title is attributed to Golden et al. (1972). Other versions of VRP emerged in the early 1970s; e.g. fleet routing (Levin 1971), dial-a-bus systems (Wilson and Sussman 1971), transportation network design (O’Connor and DeWald 1970), routing of public service vehicles (Marks and

Stricker 1970), distribution management (Eilon et al. 1971), and solid waste collection (Liebman 1970). Probabilistic content was introduced to the VRP by Golden and Stewart (1978). Solomon (1983) added time-window constraints to the classical VRP and introduced a set of well known benchmark problems now known as "Solomon Instances." During the 1980s, VRP research generated different static configurations of the original problem. Due to computational complexity and scarcity of microcomputers, stochastic and dynamic versions of the VRP were not much studied. For a further inquiry into major problem types, formulations and solution methods of this era, one may refer to the works of Laporte and Nobert (1987), Assad (1988), and Laporte (1992).

VRP research accelerated during the 1990s. Primarily due to microcomputer capability and availability, researchers could develop and implement more complex search algorithms. During this era the term *meta-heuristics* was introduced to name a number of search algorithms for solving these VRPs as well as other combinatorial optimization problems. Gendreau et al. (1998) studied VRP applications with meta-heuristics such as: 1) Simulated Annealing, 2) Deterministic Annealing, 3) Tabu Search, 4) Genetic Algorithms, 5) Ant Systems, and 6) Neural Networks. For further information one may refer to works of Wasserman (1989), Osman and Kelly (1996), Osman and Laporte (1996), Dorigo et al. (1996), Aarts and Lenstra (1997), Glover and Laguna (1997), and Cordeau et al. (2002).

In the database we compiled, the first stochastic VRP (SVRP) dates back to Cook and Russell (1978) – a simulation study. Although a few papers appeared in the 1980s on SVRP, the pace quickened in this field during the 1990's. Developments in computer science and technology helped researchers model and solve SVRPs easily. For a review of SVRP, see Gendreau et al. (1996b) and Yaohuang et al. (2002); for solution algorithms see Roberts and Hadjiconstantinou (1998) and Park and Hong (2003). The first dynamic VRP study dates back to Powell (1986). Together with the boasting CPU capabilities and the enhancements in vehicle tracking, data storage, and exchange media, dynamic VRPs (DVRP) became more frequent in the literature during the second half of the 1990s. For a deeper inquiry in DVRP one may refer to studies by Psaraftis (1995), Gendreau and Potvin (1998), Savelsbergh and Sol (1998), Larsen (2000), Larsen et al. (2002), and Fleischmann et al. (2004).

2.1.3. A Review of Some Previous Classification Efforts and Taxonomies

As discussed in Reisman (1992), there are at least two efficient and effective ways of consolidating knowledge. One of these is to create a taxonomy and the other is to create a generalized framework (a general model or theory) that subsumes all existing models, facts, or theories within that field. A taxonomy displays the subject's domain in terms that are easy to understand, communicate, teach, learn, and work with. It enables efficient and effective classification of any and all contributions/publications. In turn, this enables efficient and effective storage, recall, sorting, and/or statistical analyses. Because such classification results are meaningfully machine readable, they enable further meta-research which includes, but is not limited to, identification of voids in the literature and, therefore, directions/specifications for research to be performed. As shown in Vogel and Weterbe (1984) and Reisman (1992), there are many other uses for such a classification scheme. Bibliometric analysis is one of the more important of these for graduate OR/MS and Decision Science education.

Some taxonomical efforts about VRPs have appeared in the literature. Bodin (1975), first to introduce a taxonomical structure, focused on routing and scheduling problems with static requirements. Bodin provided a classification scheme with the aim of offering solution methods for each subcategory identified. Bodin and Golden (1981) extended Bodin's work by including a hierarchical model that encompasses very simple vehicle scheduling problems to complex scheduling-routing problems. This study is the first in providing a detailed list of problem attributes used in the literature. These two studies were later extended by Min et al. (1998) to provide a taxonomy with a perspective in location-allocation-routing problems. The authors, however, focused on static problems and justified this limited scope by the scarcity of dynamic problems. They also introduced a classification model for solution methods. However, their taxonomy failed to provide any insight into meta-heuristics. The model cannot address today's more complex problem attributes, either. Desrochers et al. (1990) introduced a three-level classification scheme that is simple, yet elaborate, and robust enough to represent the contemporary literature at the time of its publication. At the first level of this classification, the problems are studied based on "*addresses*" (the network's node characteristics), "*vehicles*" (characteristics of the vehicles and their routes), "*problem characteristics*" (the underlying network, service strategy, and constraints on relations between addresses and vehicles), and "*objectives*" (the objective function's characteristics). The Desrochers et al. (1990) model was applied to different problems within the literature. However, their taxonomy lacks the ability to incorporate current stochastic and dynamic VRPs as well as other static routing problems with uncommon constraints. Their work is practical and represents a good model for an academic standard in taxonomy. In a later study Desrochers et al. (1999) developed a model representation and algorithm management system for vehicle routing, as well as for scheduling problems. The management system proposed by Desrochers et al. (1999) is most helpful in handling real-world problems, from observation to formulation, algorithm selection, and ultimately to implementation. Current and Marsh (1993) provided a classification for the multiobjective design of transportation networks within three hierarchical levels. The first level of the taxonomy partitions the multiobjective network problems into eight branches, one of which is the VRP. Each branch is divided into two sub-groups based on whether it incorporates a heuristic or exact algorithm as a solution method. Although the taxonomy can handle many aspects, especially those related to the objective function, it provides hardly any information about constraints, network properties, or other physical attributes of problems. However, major contribution of the introduced model is that it clearly identifies and isolates the vehicle routing problem among a series of similar problem types with commonalities. Powell and Shapiro (1999) provided a more detailed classification and a representational structure. Their focus was on "dynamic resource transformation problems." This wide class further divides into "dynamic resource allocation problems," "dynamic resource scheduling problems," and "dynamic resource management problems." The study comprised, but was not limited to, vehicle routing, scheduling, and dispatching problems. They listed a number of problem types available in the literature and sorted them according to increasing levels of complexity and introduced a representational template with three major fields: *information, process,* and *controls*. However, with its level of detail, the paper fails to address many versions and configurations of the VRP. Actually, the authors did not attempt further development of their template into a single robust taxonomic structure. Yet, with its current state, this study helps enclose the vehicle routing problem among a series of related problem types, some of which are versions of the VRP itself. One important review and taxonomical study was provided by Psaraftis (1995). After identifying currently available dynamic transportation models, he provided a non-exhaustive classification of distribution problems under ten configurations occurring in real world environments. Psaraftis clearly defined dynamic problems and isolated them from the static ones. Since a dynamic nature comes with input data characteristics, he provided a taxonomy useful in characterizing input information attributes. After defining twelve major factors that distinguish between static and dynamic transportation models, Psaraftis suggested some future research directions.

In addition to the taxonomy-based studies summarized above, two reviews need to be mentioned. After identifying what makes VRPs stochastic, Gendreau et al. (1996b) discussed solution concepts. The study further delineated stochastic VRP groups and discussed their configuration by citing implemented solution methods found in the literature. Returning to the original routing problems, Laporte and Osman (1995) provided a bibliography of 500 studies. They group the problems into four major categories based on configuration: the traveling salesman problem (TSP), the VRP, the

Chinese postman problem, and the rural postman problem. Their study provided further detail for these problem types and provided characteristics reported in the literature. Laporte and Osman's study is one of a kind in the number of bibliographical entities provided on the vehicle routing literature.

2.1.4. VRP in Transportation Models

Although a concise and robust definition such as the one above delineates the general boundaries of the VRP, it is a necessity to clearly isolate this problem type and other similar problem types in the general family of transportation network models. An effort is needed since researchers need to know what constitutes the VRP and what is considered to be inside, as well as outside of it. The partitioning of the network models will help clearly set the boundaries of the research and exclude analogous but less relevant other transportation models. Such a partitioning of the literature was first proposed by Current and Marsh (1993) Their high level framework lists VRPs together with seven other multi-objective transportation network problems which are shortest path problem, transportation problem, assignment problem, transshipment problem, optimal network design problem, spanning tree problem, and network flow problems. As seen, this proposed partitioning does not simply deal with route design but a rather broader context based on the underlying mathematical structure and/or the purpose of the formulation. Thus, the Current and Marsh (1993) taxonomy dealt with a universe larger than its VRP constellation.

Another high level classification scheme for the general transportation models was introduced by Psaraftis (1995). Although dynamic configurations are addressed, elimination of the word "dynamic" makes the proposed classification hold for the static models as well. Psaraftis (1995) offers the following six categories under transportation models: 1) vehicle routing, 2) shortest path, 3) traffic assignment, 4) fleet management, 5) air-traffic control, and 6) facility location models. Rather than providing further detail, this classification identifies all versions of the vehicle routing under a single category.

Bodin et al. (1983) classified the routing problems into seven categories: 1) simple TSP, 2) multi-travel salesman problem, 3) single-service station with multi-vehicle routing problems, 4) multi-service station with multi-vehicle routing problems, 5) single service station with random demand multi-vehicle routing problems, 6) Chinese postman problem, and finally, 7) Chinese postman problem with load constraints. However, this classification is not coherent, i.e. it does not describe each category with identical level of detail. The classification above may briefly be unified under 3 categories: TSP, VRP, and the Chinese postman problem, avoiding further detail.

In this study, a detailed but not exhaustive list of subcategories for the generalized routing problem is proposed as 1) shortest path problem, 2) Chinese postman problem, 3) rural postman problem, 4) dial-a-ride service route problem, 5) arc routing problem, 6) TSP, and 7) VRP. Figure 2.2 is a visual sketch for the classification above. This figure is meaningful in the sense that it presents a family of routing problems encompassing the VRP and provides a sense of borders that isolate the VRP from all else, which differ by the routing scenario, constraints implied, and thus the imposed mathematical modeling requirements.

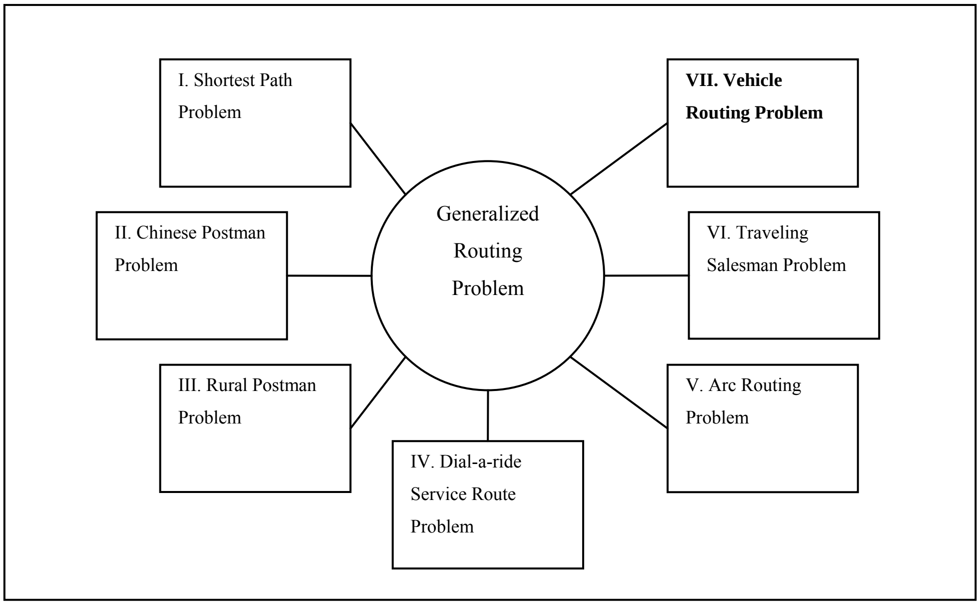

Figure 2.2 Generalized routing problem and its basic sub-categories

This second chapter's aim and scope is confined to identifying configuration characteristics and basic design parameters of the VRPs. The focus area is contained and isolated from other routing problems given above in the categorization. The taxonomy lists and accommodates the design specifications encountered so far in the literature, however while compiling those, some characteristics observed at other routing problems transferable to VRP are also included. It is rather convenient to study each problem after fitting it into the model. As the literature reports more disparate problems, the proposed taxonomy will make it easy to identify the similarities and the differences as well as accommodate new attributes in its structure.

2.2. Epistemology of the Literature

2.2.1. Literature Search Process

At the outset, a wide set of academic studies, databases such as *EBSCO Inspec, Ei Compendex,* and *ISI Web of Science*, and the bibliographical list provided by Larsen (2003) were utilized to compile information on the VRP. The databases were search on 25 January 2006, using "vehicle routing" as the search phrase. This exact phrase was searched in "Subject/Title/Abstract" field options. This constraint reduced the number of hits irrelevant to this study and eliminated those hits where VRP was tangential, or referred to, but was not studied as the main topic. Additionally, bibliographical entries that refer to studies in languages other than English were eliminated.

2.2.2. Statistical Findings

The 1475 bibliographical entities, which included journal articles, books, book chapters, technical reports, and articles from various conference proceedings, were kept after an initial review which eliminated the articles which are related with other topics rather than the VRP itself. Table 2.1 provides a breakdown of the compiled bibliography.

Table 2.1 Listing of different types of studies in the VRP literature

Entity Type	**Count**
Journal Article	1011
Proceeding	372
Technical Report	61
Book	5
Book Chapter	26
Total	**1475**

Although the Mitchell Memorial Library has full access to most of the above 1011 journal articles, some of them are either not accessible or only the abstract is accessible. The main reasons for this inaccessibility are geographical because access to regional journals is blocked and publisher databases such as Springer and EBSCO provide only limited access. Among all the journal articles, full access to 676 and limited access to 314 is available. Only bibliographical information can be accessed in the rest of the remaining twenty one articles. Those twenty one journal articles appeared in journals that are mostly old, discontinued, or local to a scientific association. For instance, "Operations Research Quarterly" is the most frequent source in this group of twenty one journal articles. The six articles that are appeared in "Operations Research Quarterly" have been published between 1969 and 1976.

2.2.3. Literature Growth

As indicated above, introduction of microcomputers has enabled researchers to solve VRP type combinatorial optimization problems more efficiently. As the computing power has increased, researchers and practitioners were able to solve ever larger VRP problems. As a result of this the number of articles published in this area during the

1980s and mostly in the 1990s has increased significantly. During the 1990s, computational methods to obtain quality solutions led to the development of new fields such as meta-heuristics and fast, real-time heuristics, which require high programming skills. Tabu search, evolutionary algorithms, and other search algorithms emerged and have been widely applied on the VRP during the last decade. Figure 2.3 shows the number of VRP articles published since the early 1950s. As can be seen, there is a rising trend in the number of articles published in this field.

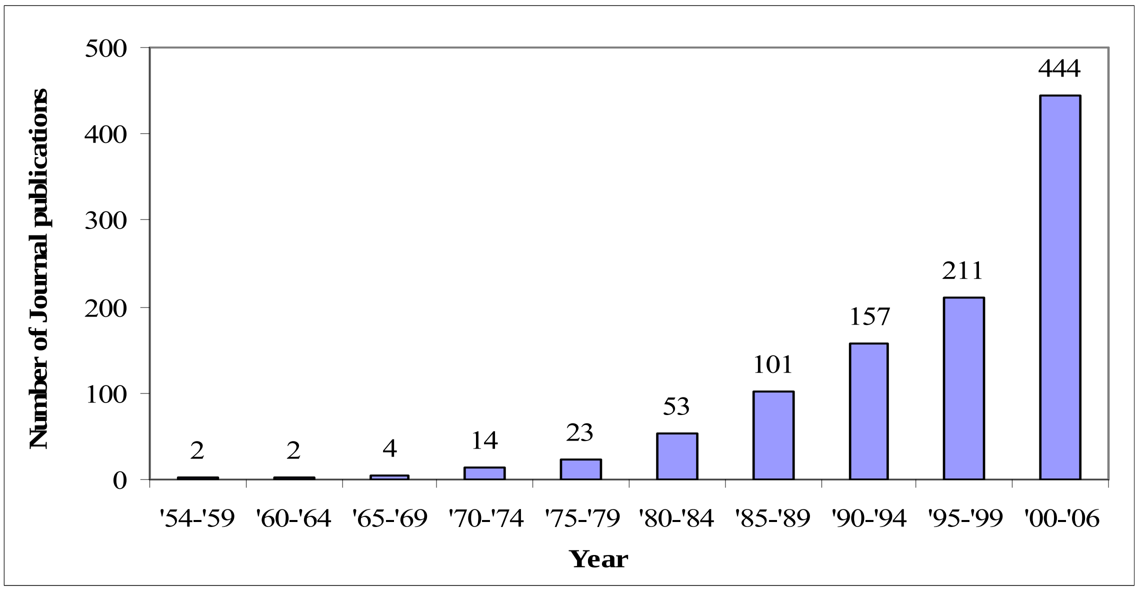

Figure 2.3 Number of VRP articles published in refereed journals from 1954 to 2006

The names of the most frequently appearing researchers within the VRP field are Gilbert Laporte (66 times), Michel Gendreau (42 times), Jean-Yves Potvin (41 times), Bruce L. Golden (34 times), Marius M. Solomon (22 times), and Christos D. Tarantilis (19 times). According to Ei Compendex database search results, some of the most frequent keywords that appear along with the phrase "vehicle routing" are *heuristic* *methods, algorithms, problem solving, optimization, transportation routes, mathematical models,* and *scheduling*. Most frequent classification codes given by the same database for studies on VRP are *applied mathematics, optimization techniques, highway transportation, management, computer software data handling and applications, operations research, artificial intelligence, numerical methods,* and *combinatorial mathematics*.

Due to enhanced data collection, tracking capabilities, and computational availabilities, trends in the VRP literature have shifted from static to more dynamic cases that incorporate real-time data. Since most of the characteristics of dynamic problems are also common to the static case, this paper provides a taxonomy of all VRP publications. Figure 2.4 shows the accumulation of VRP refereed articles on semi-logarithmic coordinates. If one eliminates the first two years of data, i.e. the gestation period, as well as the ongoing year 2006 the result is almost a straight line time-trend described by

$$\textit{Number of Publications} = ce^{0.0609t}$$

In other words, the literature growth is almost perfectly exponential with a 6.09% annual growth rate. This fact alone demonstrates VRP's vitality. However, it is not as rapid when compared to other contemporaneous OR/MS growth disciplines, e.g. Data Envelopment Analysis (DEA) having a rate of 25.5%, Flowshop Scheduling with 15.1%, Cell Manufacturing at 10.6 % (Gattoufi et al. 2004b) and Mass Customization with 10.6% (Kumar et al. 2006). The slower rate of increase may be explained in several ways: perhaps solutions of VRP problems require much more sophistication than do the others. This is certainly the case with DEA and Mass Customization.

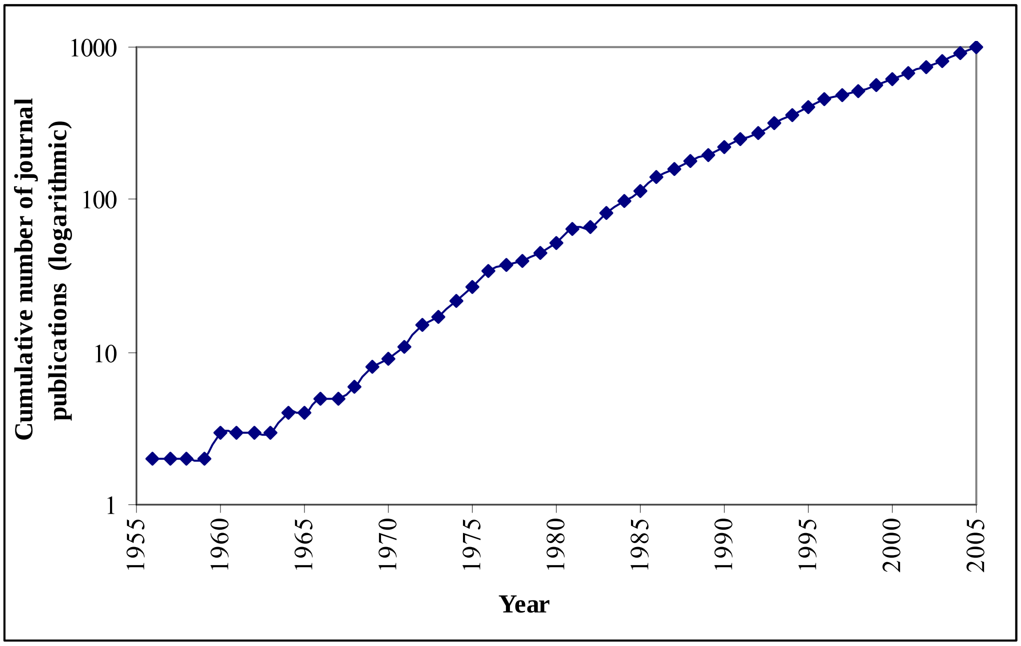

Figure 2.4 Cumulative number of VRP articles for the period 1956-2005

2.2.4. Major Article Producers and the Journals of Choice

In Table 2.2 the journals are arrayed in descending order of the total number of VRP articles published. From this it can be seen that *EJOR* and *Transportation Science* are by far the journals of preference for VRP authors. *EJOR* and *Transportation Science* together account for 20% of all VRP articles published in refereed journals. It can further be seen that the top five journals in Table 2.2 account for 41% of the literature, the top ten journals for almost 52%, and finally twenty eight journals constitute 75% of all the journal entities. All other journals have no more than five hits each. Moreover with the exception of the seven articles which appeared in the *Journal of Food Engineering* the publications are dominated by what might be considered the "hard core" of OR/MS literature. If such is indeed the case then this fact alone may not bode well for the field as will be discussed later.

Among the articles published in refereed proceedings, fifty five of them appeared in Lecture Notes in Computer Science, twenty four of them in Transportation Research Record, and ten of them in Lecture Notes in Artificial Intelligence. The total figure of these articles is eighty nine and it constitutes 24% of the 372 proceeding entries we identified. These articles are not counted with the journal articles since they were classified as proceedings in the databases where we performed our search.

Table 2.2 Listing of articles with respect to academic journals

Journal Title	***Count***
European Journal of Operational Research	116
Transportation Science	89
Journal of the Operational Research Society	77
Computers & Operations Research	69
Operations Research	61
Networks	35
Annals of Operations Research	31
Transportation Research Part B: Methodological	19
Interfaces	18
Journal of Heuristics	17
Computers & Industrial Engineering	16
Omega	14
Discrete Applied Mathematics	13
Management Science	13
Mathematical Programming	13
Operations Research Letters	13
ORSA Journal on Computing	13
Asia-Pacific Journal of Operational Research	12
INFORMS Journal on Computing	12
International Transactions in Operational Research	12
OR Spektrum	10
American Journal of Mathematical and Management Sciences	9
INFOR	9
Transportation Research Part A: Policy and Practice	9
Journal of Food Engineering	7
Decision Support Systems	6
Transportation Planning and Technology	6
Transportation Research Part E: *Logistics and Transportation Review*	6
First 5 Total	***412***
First 10 Total	***532***
Total of first 28	***725***
Others	***286***
Total	***1011***

Over the years, three mainstream journals, *EJOR*, *Transportation Science* and *JORS*, dominated the VRP literature. These journals combine to create a ‘footprint’ rate (percentage of VRP articles published in these journals as compared to the total VRP publications) of 35% from 1983 to 1985. Although this rate fell down to 16% in 1986, the figure peaked at 47% and 45% consecutively in 1988 and 1989. The average of the succeeding years through 2005 is around 29% which is only one more point than the overall footprint rate of these three journals, 28%, among all the VRP literature we have collected. The highest number of VRP articles published in a single year in *EJOR* was fifteen in 2004. The highest for *JORS* was fourteen in 2002, and *Transportation Science* had twelve in 2004. Although explaining these trends is not easy, studying the preferences of VRP researchers as well as types of studies submitted to particular journals could explain their motivations. One thing is certain: editorial boards and policies play a key role in defining the "*coloration,*" the mix of articles each journal publishes. Figure 2.5 gives the cumulative distribution of percentage of articles published with respect to the percentage of all journals we discovered in our search.

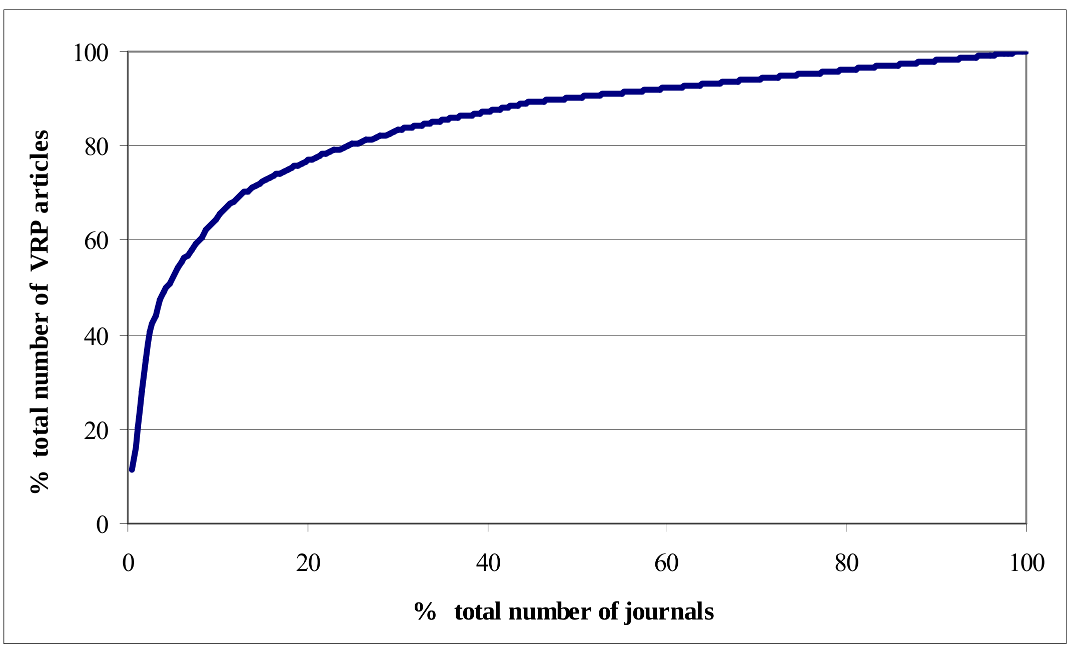

Figure 2.5 Cumulative percentage of articles per journal

2.3. Need for a Taxonomy

The size and growth rate of the VRP literature demands a systematic way to classify the various contributions in a manner that will vividly provide a panoramic view of what exists and will also clearly identify any existing gaps in the state of the art as suggested by Reisman (1992, 1993). Hence VRP literature needs a taxonomy which:

> Graphically, symbolically or both, will vividly display the similarities and the differences among the various contributions, thus demonstrating the relationship of all contributions and the practical applications of each to other. It will provide a framework by which all of the existing knowledge can be systematically filed and therefore recalled efficiently and effectively. By providing what amounts to an aerial view -a picture of the territory- it will identify the voids in the literature… Knowledge consolidation is a means to various ends, and it is also an end to itself. It is a means toward the end of more efficient and more effective teaching and learning of new or existing knowledge. It is a means toward the end of more efficient storage and more effective recall and/or retention of knowledge. It is a means toward a more efficient and more effective processes of research leading to the yet unknown, to the design of the yet unavailable, and it is a means toward more efficient problem solving – (Reisman 1992).

Moreover,

> The key to taxonomy effectiveness rests on criteria of comprehensiveness, parsimony and usefulness. Obviously, to be effective, a taxonomy must represent the full spectrum of the research chosen for categorization. Thus, comprehensiveness is a necessary condition for effectiveness. It is, however, not sufficient. To further be effective, a taxonomy should be parsimonious. It should not include unnecessary categories. Finally, to be considered effective, the taxonomy should be robust and generally useful. The categories should be reasonably if not mutually exclusive, i.e., non-overlapping, reasonably distinct, meaningful, commonplace, and descriptive to allow utilization by a wide variety of interested persons – (Vogel and Weterbe 1984).

A taxonomy is not only a tool for systematic storage, efficient and effective teaching/learning, and recall for usage of knowledge, but it is also a neat way of pointing to knowledge expansion and building. It identifies voids, potential theoretical increments or developments, and potential applications for the existing theory. Basic motivations and uses for taxonomy may be listed as follows:

1. It defines or delimits the boundaries of a subject domain and that is, in itself, useful information.

2. It vividly, efficiently, and effectively shows/displays all of that domain's attributes/dimensions.

3. It vividly, efficiently, and effectively shows/displays that any one of the possible combinations of these attributes/dimensions defines or delimits the boundaries of a subject sub-domain.

4. It allows one to organize his or her knowledge about the domain, and this has major implications for teaching, learning, storing, and recalling information.

5. It allows one to have a panoramic view of the entire "forest" while examining and classifying a given "tree."

6. It allows one to unify disjointed and disparate subfields or sub-disciplines into a meaningful whole.

7. It allows one to identify voids and well explored territories in the extant literature base, and this is very important for researchers, funding agencies and other decision makers.

Clearly, as is the case in one of the greatest and best-known taxonomies of all time, the Periodic Table of Elements (Mendeleyev 1889), what is presented here is open for incremental evolutions. A taxonomy is very much dependent on the definition of the boundaries of the universe it classifies. Hence, the classification developed in this study is open to expansion as the scope of the routing problem enlarges. With the Periodic Table of Elements (PTE) as a role model, one can discuss the usage of a taxonomy to knowledge building. The PTE has always indicated cells which described with great efficiency elements yet to be discovered. Thus if and when the taxonomy classifies all extant VRP articles, the cells remaining empty will vividly show the voids in the literature. To be sure some of those void "cells" have a greater research potential than others. However, the empty cells will all create a full set of specifications for the researchers to pursue as amply demonstrated by Reisman (1992) while the already filled cells provide guidance and methodological aid for the inquirers in this field. Professionals and researchers may easily identify which study covers what aspect of the very original problem they have. If on the other hand, only a sample of the extant papers is classified, then the probability that a void is identified truly so increases with the sample's richness and representativeness.

VRP has already generated a large enough literature to allow it to be considered as a separate and distinct field of knowledge. The increasing interest in VRP makes a systematic elaboration of this field more crucial in helping current researchers as well as in attracting potential newcomers to the field.

The current attempt to define a taxonomy for the VRP literature may have its own disadvantages but it does not suffer from ambiguity as was the case for classifications by Desrochers et al. (1990 and 1999). In fact, this taxonomy may be too detailed. This makes applications cumbersome, but also increases its descriptive powers. However, aggregating sub-classifications and/or pruning outer branches is easier then the inverse. Having to disaggregate classifications already made typically requires a great deal of effort. The taxonomy proceeds in an arborescent way (Reisman 1992) as is illustrated in Figure 2.6. The cells that do not have any sub-branches in Figure 2.6 are referred to as end-nodes and will be used to classify a set of selected papers.

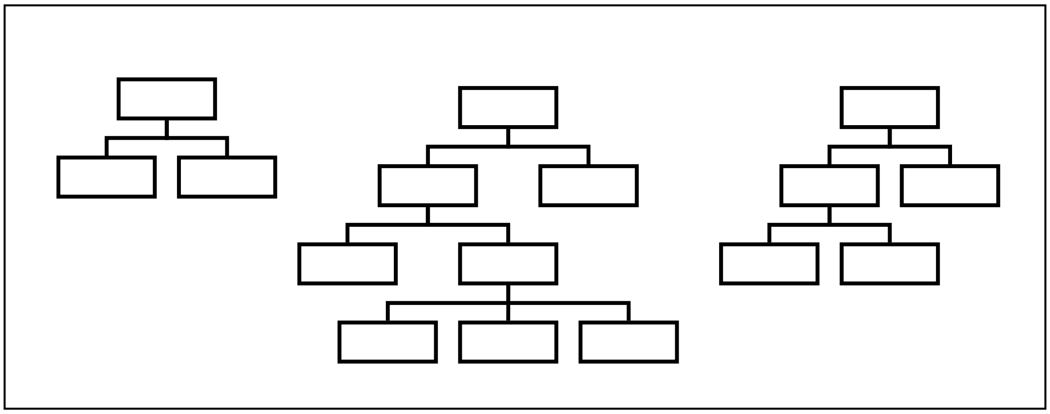

Figure 2.6 Attribute vector description based taxonomy

2.4. VRP Taxonomy

In this section the new taxonomy is presented and the main features considered while building it are briefly explained. Definitions as well as justifications for those main features are provided and some terms within the content of the taxonomy are identified.

The VRP taxonomy is built in an arborescent way. As it can be seen in Figure 2.7 the branching levels from top to bottom are, at most, three in order to provide coherence and parsimony, yet sacrifice nothing in terms of comprehensiveness. The classifications and sub-classifications at the first and second levels are not strictly distinguishing in the sense that a paper might address many different subcategories of the same category. The first partitioning is built so that each paper may be classified by five different aspects within a single level (see Figure 2.7). This first level of branching is composed of: 1) Type of Study, 2) Scenario Characteristics, 3) Problem Physical Characteristics, 4) Information Characteristics, and 5) Data Characteristics. Within the first characteristic set, Type of Study, a paper is distinguished based on its content and whether it fits one of the 4 sub-categories presented. After identifying the type of the study, for those papers that comprise a problem, the rest of the categories are introduced. In the second category, Scenario Characteristics, those factors that are not a part of the constraints embedded into the solution, but part of the problem scenario, are listed. The third category, Problem Physical Characteristics, comprises those factors that directly affect the solution. This category is an extended version of the works of Bodin (1975) and Bodin and Golden (1981). The fourth category, called Information Characteristics, deals with the solution of the VRP presented by qualifying the nature, accessibility, and processing of the information. This category is taken from the work of Psaraftis (1995), which actually aims to classify dynamic VRPs. However, we believe that the same structure may be used to study the information characteristics of all dynamic, static, and stochastic VRPs. Although the last category, Data Characteristics, possesses a similar name to the previous category, it introduces a distinctive purpose: to classify the type of data based on its origin.

1. Type of Study
   - 1.1. Theory
   - 1.2. Applied methods
     - 1.2.1. Exact methods
     - 1.2.2. Heuristics
     - 1.2.3. Simulation
     - 1.2.4. Real time solution methods
   - 1.3. Implementation documented
   - 1.4. Survey, review or meta-research

Figure 2.7 Taxonomy for the VRP Literature

2. Scenario Characteristics
   2.1. Number of stops on route
      2.1.1. Known (deterministic)
      2.1.2. Partially known, partially probabilistic
   2.2. Load splitting constraint
      2.2.1. Splitting allowed
      2.2.2. Splitting not allowed
   2.3. Customer service demand quantity
      2.3.1. Deterministic
      2.3.2. Stochastic
      2.3.3. Unknown[1]
   2.4. Request times of new customers
      2.4.1. Deterministic
      2.4.2. Stochastic
      2.4.3. Unknown
   2.5. On site service/waiting times
      2.5.1. Deterministic
      2.5.2. Time dependent
      2.5.3. Vehicle type dependent
      2.5.4. Stochastic
      2.5.5. Unknown
   2.6. Time window structure
      2.6.1. Soft time windows
      2.6.2. Strict time windows
      2.6.3. Mix of both
   2.7. Time horizon
      2.7.1. Single period
      2.7.2. Multi period
   2.8. Backhauls
      2.8.1. Nodes request simultaneous pick ups and deliveries
      2.8.2. Nodes request either linehaul or backhaul service, but not both.
   2.9. Node/Arc covering constraints
      2.9.1. Precedence and coupling constraints
      2.9.2. Subset covering constraints
      2.9.3. Recourse allowed

3. Problem Physical Characteristics
   3.1. Transportation network design
      3.1.1. Directed network
      3.1.2. Undirected network
   3.2. Location of addresses (customers)
      3.2.1. Customers on nodes
      3.2.2. Arc routing instances

Figure 2.8 (continued) Taxonomy for the VRP Literature

---

[1] The "Unknown" condition happens in the case that the information is revealed to the problem solution environment on real time, i.e. the information is not revealed to the system before the solution method or simulation is executed.

- 3.3. Geographical location of customers
  - 3.3.1. Urban (scattered with a pattern)
  - 3.3.2. Rural (randomly scattered)
  - 3.3.3. Mixed
- 3.4. Number of points of origin
  - 3.4.1. Single origin
  - 3.4.2. Multiple origins
- 3.5. Number of points of loading/unloading facilities (depot)
  - 3.5.1. Single depot
  - 3.5.2. Multiple depots
- 3.6. Time window type
  - 3.6.1. Restriction on customers
  - 3.6.2. Restriction on roads
  - 3.6.3. Restriction on depot/hubs
  - 3.6.4. Restriction on drivers/vehicle
- 3.7. Number of vehicles
  - 3.7.1. Exactly *n* vehicles (*TSP in this segment*)
  - 3.7.2. Up to *n* vehicles
  - 3.7.3. Unlimited number of vehicles
- 3.8. Capacity consideration
  - 3.8.1. Capacitated vehicles
  - 3.8.2. Uncapacitated vehicles
- 3.9. Vehicle homogeneity (Capacity)
  - 3.9.1. Similar vehicles
  - 3.9.2. Load specific vehicles (each vehicle can be used to handle specific types of loads)
  - 3.9.3. Heterogeneous vehicles
  - 3.9.4. Customer specific vehicles (A customer must be visited by a specific type of vehicle)
- 3.10. Travel time
  - 3.10.1. Deterministic
  - 3.10.2. Function dependent (a function of current time)
  - 3.10.3. Stochastic
  - 3.10.4. Unknown
- 3.11. Transportation cost
  - 3.11.1. Travel time dependent
  - 3.11.2. Distance dependent
  - 3.11.3. Vehicle dependent (Cost of operating a vehicle is not negligible)
  - 3.11.4. Operation dependent
  - 3.11.5. Function of lateness
  - 3.11.6. Implied hazard/risk related

4. Information Characteristics
- 4.1. Evolution of information
  - 4.1.1. Static
  - 4.1.2. Partially dynamic
- 4.2. Quality of information
  - 4.2.1. Known (Deterministic)
  - 4.2.2. Stochastic
  - 4.2.3. Forecast
  - 4.2.4. Unknown (Real time case)

Figure 2.9 (continued) Taxonomy for the VRP Literature

4.3. Availability of information
  4.3.1. Local
  4.3.2. Global
4.4. Processing of information
  4.4.1. Centralized
  4.4.2. Decentralized

5. Data Characteristics
  5.1. Data Used
    5.1.1. Real world data
    5.1.2. Synthetic data
    5.1.3. Both real and synthetic data
  5.2. No data used

Figure 2.10 (continued) Taxonomy for the VRP Literature

2.5. Testing the Taxonomy with Disparate VRP Articles

By using a group of articles that represent rather different approaches and that address different issues of the VRP, the taxonomy of Figure 2.7 is tested for its robustness and its ability to discriminate in a parsimonious manner. The articles used for that purpose are identified in Tables 2.3 and 2.4. Note that in applying the above taxonomy to classify a specific document, some cells may remain empty. This means that the paper does not address or involve that cell's attributes. The domains or attributes corresponding to end-nodes are marked with '**X**.' Shaded columns represent domains or classes which branch, so that shading suggests why these columns are not marked. This representation scheme enables us to assign more designations in a confined space.

As can be seen, Tables 2.3 and 2.4 have many blank cells. However, these classifications for the papers that test the taxonomy leave only a small percentage, 1.23%, of the end-nodes in each tree empty. For the total number of columns, nodes marked only once constitute 4.94%. This value becomes 12.34% for nodes marked twice and 6.17%

for nodes marked thrice. These percentages clearly attest to the fact that the taxonomy of Figure 2.7 is robust enough to succinctly classify the large diversity of the VRP literature. Of the end-node classifications, the only empty column denotes ‘vehicle type dependent service/waiting times’ which was not addressed in any paper tested. By checking the rest of the bibliographic list, it may be concluded that this attribute represents work not previously attempted by anyone, rather than a superfluous node within the current paradigm of VRP’s scope.

Attributes which are marked less than thrice in the figure are: 2.3.3, 2.4.3, 2.5.2, 2.5.4, 2.5.5, 2.8.1, 3.2.2, 3.8.2, 3.9.2, 3.9.4, 3.10.3, 3.10.4, 3.11.6, 4.2.3, 4.4.2, and 5.2. Among these attributes, 2.3.3, 2.4.3, 2.5.5, and 3.10.4 denote unknown data observed in real-time. These VRPs with dynamic content have been relatively less studied or reported in archival journals thus far.

Table 2.3 Summary of the illustrative classifications (attributes 1 through 3.3)

Article title	1.	1.1.	1.2.	1.2.1.	1.2.2.	1.2.3.	1.2.4.	1.3.	1.4.	2.	2.1.	2.1.1.	2.1.2.	2.2.	2.2.1.	2.2.2.	2.3.	2.3.1.	2.3.2.	2.3.3.	2.4.	2.4.1.	2.4.2.	2.4.3.
Toth, Vigo (1999)					X							X				X		X				X		
Bent, van Hentenryck (2001)					X							X				X		X						
Laporte et.al. (1999)					X				X			X			X			X						
Rego, Roucairol (1995)					X								X			X			X				X	
Soumia et.al. (2000)						X	X						X			X							X	
Savelsbergh, Sol (1998)						X	X						X			X				X			X	
Gendreau et.al. (1996a)					X								X			X			X					
Nagy, Salhi (2005)					X				X			X				X		X				X		
Nanry, Barnes (2000)					X							X			X			X				X		
Fleischmann et.al. (2004)							X	X					X			X		X						X
Meng et.al. (2005)				X								X				X								
Jaw et.al. (1986)					X							X										X		
Letchford and Eglese (1998)		X		X								X				X						X		
Eglese (1994)					X							X												
Lee and Ueng (1999)					X							X				X		X				X		
Brandao and Mercer (1997)					X							X				X		X				X		
Angelelli and Speranza (2002)					X							X				X		X				X		
Cordeau, Laporte (2001)					X							X				X		X				X		
Archetti et.al. (2005)		X										X			X			X				X		
Fu (2002)		X			X							X				X		X				X		
Chuah and Yingling (2005)				X	X							X				X				X				X
Frizzell and Griffin (1995)					X							X			X				X			X		
Zagrafos and Androutsopulos (2004)				X								X				X		X				X		
Kim et.al. (2005)		X				X	X					X										X		
Trudeau and Dror (1992)					X	X						X				X			X				X	
Malandraki and Daskin (1992)				X	X				X			X				X		X				X		
Sural and Bookbinder (2003)				X								X				X		X				X		
Bard et.al. (1998)				X	X							X			X			X				X		
Ahn and Shin (1991)					X							X										X		
Jaillet (1988)		X											X									X		

Article title	2.5.	2.5.1.	2.5.2.	2.5.3.	2.5.4.	2.5.5.	2.6.	2.6.1.	2.6.2.	2.6.3.	2.7.	2.7.1.	2.7.2.	2.8.	2.8.1.	2.8.2.	2.8.3.	2.9.	2.9.1.	2.9.2.	2.9.3.	3.	3.1.	3.1.1.	3.1.2.	3.2.	3.2.1.	3.2.2.	3.3.	3.3.1.	3.3.2.	3.3.3.
Toth, Vigo (1999)																X			X						X		X				X	
Bent, van Hentenryck (2001)										X															X		X					
Laporte et.al. (1999)																									X		X					
Rego, Roucairol (1995)									X										X						X		X					X
Soumia et.al. (2000)								X																	X		X			X	X	X
Savelsbergh, Sol (1998)										X									X						X		X				X	
Gendreau et.al. (1996a)																									X		X				X	
Nagy, Salhi (2005)															X	X									X		X					
Nanry, Barnes (2000)		X								X									X						X		X					
Fleischmann et.al. (2004)						X		X											X					X			X			X		
Meng et.al. (2005)		X	X						X			X							X						X							
Jaw et.al. (1986)									X			X				X			X						X		X				X	
Letchford and Eglese (1998)		X							X				X								X				X			X				
Eglese (1994)									X			X													X			X		X		
Lee and Ueng (1999)		X										X													X		X					
Brandao and Mercer (1997)		X							X			X													X		X					
Angelelli and Speranza (2002)													X							X	X			X			X				X	
Cordeau, Laporte (2001)		X								X		X													X		X					
Archetti et.al. (2005)												X													X		X					
Fu (2002)									X			X					X		X						X		X			X		
Chuah and Yingling (2005)										X			X												X		X					
Frizzell and Griffin (1995)					X					X			X								X				X		X			X		
Zagrafos and Androutsopulos (2004)		X							X			X				X									X		X				X	
Kim et.al. (2005)								X				X												X			X					
Trudeau and Dror (1992)					X			X					X							X	X				X		X			X		
Malandraki and Daskin (1992)		X						X				X												X			X			X		
Sural and Bookbinder (2003)												X				X				X					X		X					
Bard et.al. (1998)		X										X				X					X				X		X					X
Ahn and Shin (1991)		X							X			X													X		X					
Jaillet (1988)												X								X					X		X					

Table 2.4 Summary of the illustrative classifications (attributes 3.4 through 5.2)

Article title	3.4.	3.4.1.	3.4.2.	3.5.	3.5.1.	3.5.2.	3.6.	3.6.1.	3.6.2.	3.6.3.	3.6.4.	3.7.	3.7.1.	3.7.2.	3.7.3.	3.8.	3.8.1.	3.8.2.	3.9.	3.9.1.	3.9.2.	3.9.3.	3.9.4.
Toth, Vigo (1999)		X			X										X		X			X			
Bent, van Hentenryck (2001)		X			X			X							X		X			X			
Laporte et.al. (1999)		X			X								X				X			X			
Rego, Roucairol (1995)			X			X		X			X				X		X					X	
Soumia et.al. (2000)		X			X				X				X										
Savelsbergh, Sol (1998)			X			X		X						X			X				X	X	
Gendreau et.al. (1996a)		X			X								X				X			X			
Nagy, Salhi (2005)		X	X		X									X			X			X			
Nanry, Barnes (2000)		X			X								X				X			X			
Fleischmann et.al. (2004)			X			X		X						X			X			X			
Meng et.al. (2005)		X				X		X	X				X										
Jaw et.al. (1986)			X			X		X						X			X					X	
Letchford and Eglese (1998)		X			X			X					X					X					
Eglese (1994)			X		X				X						X		X			X			
Lee and Ueng (1999)		X			X									X			X					X	
Brandao and Mercer (1997)		X			X			X			X				X		X					X	X
Angelelli and Speranza (2002)		X				X					X		X				X			X			
Cordeau, Laporte (2001)		X			X			X		X			X				X					X	X
Archetti et.al. (2005)		X			X										X		X					X	
Fu (2002)		X			X			X							X		X					X	
Chuah and Yingling (2005)		X			X			X		X	X		X				X			X			
Frizzell and Griffin (1995)		X			X			X		X					X		X			X			
Zagrafos and Androutsopulos (2004)		X			X			X		X	X				X		X			X			
Kim et.al. (2005)								X															
Trudeau and Dror (1992)		X			X			X		X			X				X			X			
Malandraki and Daskin (1992)		X			X			X					X				X			X			
Sural and Bookbinder (2003)		X			X								X				X						
Bard et.al. (1998)		X				X					X			X			X			X			
Ahn and Shin (1991)		X			X			X		X													
Jaillet (1988)		X			X								X					X					

Article title	3.10.	3.10.1.	3.10.2.	3.10.3.	3.10.4.	3.11.	3.11.1.	3.11.2.	3.11.3.	3.11.4.	3.11.5.	3.11.6	4.	4.1.	4.1.1.	4.1.2.	4.2.	4.2.1.	4.2.2.	4.2.3.	4.2.4.	4.3.	4.3.1.	4.3.2.	4.4.	4.4.1.	4.4.2.	5.	5.1.	5.1.1.	5.1.2.	5.1.3.	5.2.
Toth, Vigo (1999)		X						X							X			X								X					X		
Bent, van Hentenryck (2001)		X						X							X			X						X		X					X		
Laporte et.al. (1999)		X						X							X			X														X	
Rego, Roucairol (1995)		X					X	X	X							X			X					X		X						X	
Soumia et.al. (2000)		X						X			X					X			X					X		X					X		
Savelsbergh, Sol (1998)		X						X	X	X						X					X		X			X	X				X		
Gendreau et.al. (1996a)		X						X								X					X		X			X	X				X		
Nagy, Salhi (2005)		X						X							X			X						X		X					X		
Nanry, Barnes (2000)		X					X				X				X			X						X		X					X		
Fleischmann et.al. (2004)					X		X				X					X				X	X			X		X				X			
Meng et.al. (2005)			X				X	X							X			X						X		X						X	
Jaw et.al. (1986)		X					X				X				X			X						X		X					X		
Letchford and Eglese (1998)		X								X					X			X						X		X					X		
Eglese (1994)			X					X	X						X			X						X		X						X	
Lee and Ueng (1999)		X					X								X			X						X		X					X		
Brandao and Mercer (1997)		X					X	X	X						X			X						X		X				X			
Angelelli and Speranza (2002)		X						X							X			X						X		X					X		
Cordeau, Laporte (2001)		X					X								X			X						X		X					X		
Archetti et.al. (2005)		X						X	X						X			X					X			X							X
Fu (2002)			X	X			X		X		X				X				X					X		X					X		
Chuah and Yingling (2005)		X						X		X					X			X						X		X						X	
Frizzell and Griffin (1995)		X					X		X	X					X			X	X					X		X					X		
Zagrafos and Androutsopulos (2004)		X						X				X			X			X						X		X					X		
Kim et.al. (2005)					X		X		X		X					X			X	X	X			X		X						X	
Trudeau and Dror (1992)		X						X		X	X					X		X	X					X		X				X			
Malandraki and Daskin (1992)			X					X							X			X	X					X		X					X		
Sural and Bookbinder (2003)		X						X		X					X			X						X		X					X		
Bard et.al. (1998)		X						X							X			X						X		X					X		
Ahn and Shin (1991)			X	X			X								X			X	X					X		X					X		
Jaillet (1988)		X						X							X				X					X		X							X

Other less frequently observed attributes are: 2.8.1, 3.2.2, 3.8.2, and 3.11.6. These are not very common in the VRP literature or in the path selection problems studied. Routing with simultaneous backhauls (2.8.1) is a fairly new field. Since this search is confined to VRPs rather than to arc-routing, studies that involve arc-routing (3.2.2) are mostly omitted. Routing with uncapacitated vehicles (3.8.2) is a very common problem configuration that is observed in the literature, especially until early 1990s. However, after introduction of faster heuristics, more complex and realistic problems with capacitated vehicles became more common. The conventional TSP, Chinese postman, and rural postman problems are some instances resulting from no capacity constraints on the vehicles. Transportation cost calculations based on implied hazards to the environment (3.11.6) are studied in many hazardous material (hazmat) transportation problems (Erkut et al. 2005). Since most of the hazmat literature on routing focuses on the shortest path selection rather than on forming complete tours, implied hazard assessment problems within the VRP literature are relatively rare. For a recent and elaborate review on hazmat transportation one may refer to Luedtke and White (2002) and Erkut et al. (2005).

Employing load-specific (3.9.2) or customer-specific (3.9.4) vehicles are very rarely observed in the VRP literature. Traveling repairmen problems are mostly customer-specific (3.9.4) instances, while distribution of multi-commodity loads are load-specific (3.9.2) types of problems. These two problems suggest some open fields to be studied in the future.

Usage of raw forecast data (4.2.3) is quite rare. However, fitting collected data into some stochastic distribution functions and embedding these functions into problems is a more common approach. Thus, stochastic problems are more frequently observed than problems utilizing forecast data.

Information is processed and routes are generated mostly at a central location. However, with today's technology, distributed and parallel systems are easier to construct and maintain. Although decentralized processing of information (4.4.2) is a rare solution for most of the studies, with the increasing number of real-time solution instances, need for shorter response times will lead to more parallel processing algorithms and may even lead to each truck establishing its own path as real-time data is observed at customer nodes.

Review papers or theoretical studies usually need no data samples or sets, and they constitute a smaller percentage of the overall VRP literature. When Tables 2.3 and 2.4 were constructed, papers were selected to reflect very different attributes that are not common in the literature. Because of this emphasis on distinctive problem instances, review papers or theoretical papers without data usage are mostly omitted. The review papers we included in Tables 2.3 and 2.4 contain problem instances and solution algorithms (e.g. Malandraki and Daskin 1992; Laporte et al. 1999; Nagy and Salhi 2005). Thus, real or synthetic data are included in this study, and taxonomical papers are not incorporated into these figures. The only papers without data usage are those which are theoretical, and thus, frequency of papers without data usage (5.2) seems relatively low.

In basic terms, the purpose of any vehicle routing solution developer is given as "providing a high level of customer service while keeping the operating and investment costs as low as possible" (Partyka and Hall, 2000). This is possible only trough keeping the solution feasible, in a set bounded by constraints which are usually specific to the very instance studied. However, translating these fundamentals into products that meet the needs of fleets usually requires the problem solver develop a solution that will not fit into the classic VRP configuration, but go beyond it in terms of constraints and problem characteristics.

Partyka and Hall (2000) illustrate how diversely the needs and configurations of distribution problems differ in real world instances. Tabulated under route characteristics, these differences are grouped into six categories: which are about delivery characteristics, randomness of customer orders, driver specialization, ability to develop real time solutions, time windows, and multiple facilities. Except for some cases of driver specialization, each attribute mentioned can be mapped on our taxonomy. However, this chapter of the dissertation is focused on reflecting the literature trends to researchers and solution developers. Thus, any characteristic that can be observed in real life scenarios but no in the taxonomy may be added in the future as it appears in the literature. The introduced VRP taxonomy provides flexibility in expansion and offers an open structure that can be updated through time.

2.6. Concluding Remarks

Because of the subjectivity inherent in selecting illustrative papers, papers that represent different periods, different journals, differing paths of VRP analysis and differing research strategies were selected. The respective authors emanate from different countries, indeed from different continents.

The taxonomy presented has been demonstrated to be robust enough to subsume a diverse set of VRP articles, as well as all previously published taxonomies in this field. It is currently being used to classify all 1011 VRP papers published in refereed journals circa 2006. This exercise, when completed, will provide rather detailed specifications of research to be done - a road map for future workers - as was demonstrated for another subject domain in Reisman et al. (2006). At this time, test results of a well-selected subset of thirty very dissimilar and diverse papers are presented.

However, based on a less systematic review of the 676 full papers and 314 abstracts of additional VRP papers, the following can be said:

1. To date, the literature has not arrived at a consensus on how to define the VRP. Moreover, given any one definition, there is no unanimity as to how to address it.
2. Forming strategies to improve the performance of solution methods for the VRP appears to be lacking.
3. Most papers studied the VRP in a static manner, and dynamic treatments of the VRP are relatively recent because they started to emerge as of 1986.
4. No previous study has ever proposed a taxonomy as detailed or as comprehensive as the one presented in this paper.
5. No epistemological studies were performed on the VRP literature.

6. No previous study ever reported counts of published papers by year, by journal, and by solution methods used (see Reisman et al. 2006).
7. No other article investigated the most cited or the most influential author(s), paper(s), or journal(s). However, the journal with the most publications in any given field or the journal considered the most prestigious does not necessarily publish the paper with the greatest influence on subsequent research (Reisman and Oral 2005).
8. To date, no paper reported the kind of content analysis for VRP that was done for DEA in Gattoufi et al. (2004a and 2004b).

The taxonomy provided can be used in many other ways than those already mentioned. Using it in the teaching of VRP subject matter is one important way. The taxonomy defines the VRP domain very globally, yet very minutely. Moreover, it does so without taking much class time, as discussed in Reisman (2004).

The taxonomy in Figure 2.7 may well be too detailed than is necessary for common usage. If so, this violates the principle of parsimony. However, experience shows that aggregating data in hand is easier than not to have collected it in sufficient detail in the first place (Reisman 1992). Lastly, no taxonomy should be considered fixed for all time. It should evolve as the field it addresses evolves. Indeed, the above taxonomy can be meaningfully enhanced by redefining the subject. Over time, even the Periodic Table of Chemical Elements has been and remains in the constant state of evolution.

CHAPTER III

A REVIEW OF THE VEHICLE ROUTING PROBLEM WITH SIMULTANEOUS PICK-UP AND DELIVERIES

3.1. Introduction

The vehicle routing problem with deliveries and pickups (VRPDP) is a subset of the general Vehicle Routing Problem (VRP). The VRPDP allows either or both deliveries or pickups to be made simultaneously. Salhi and Nagy (1999) clearly identify three VRPDP problem types based on the service provided. They are:

i. VRP with backhauls (VRPB): The nodes are identified either as linehaul (i.e. nodes with deliveries that originate at the depot) or backhaul (i.e. nodes with items to be picked up that go to the main depot). The linehaul nodes are served prior to the backhaul nodes because, as Chen and Wu (2006) suggest, old truck design allowed rear-load functions only.

ii. VRP with mixed-load (VRPM or VRPBM): Side-loading of trucks made load rearrangement on board easier. Thus the need for serving linehaul customers first to free some space was no longer necessary and resulted in service routes with backhaul nodes in any sequence before the last linehaul node. The VRPM or VRPBM problem constitutes establishing routes composed of mixed linehaul and backhaul node sequences.

iii. VRP with simultaneous delivery and pick up (VRPSPD or VRPSDP): The VRPB and VRPBM settings caused inconvenience when some nodes requested both pick up and delivery service. Tour length increased because those nodes had to be visited twice, but the VRPSPD setting lets each node have a delivery and pick up service during the same stop. The VRPBM is a special case of VRPSPD since either delivery or pick up quantity at each node is defined as zero.

The VRPSPD model differs from the *m-dial-a-ride problem with non-unit capacity* in the sense that the traffic of goods between nodes other than the depot is strictly avoided. In the latter, *m* routes are created by using multi-unit capacity vehicles to accommodate transportation of items to and from the nodes that are on a route.

The three types of VRPDPs differ not only in the types of services they provide, but also in the ways they are modeled and the data sets used. The VRPB is modeled differently from the other two, but the VRPSPD and VRPBM may be modeled the same way. However, the data set used by each model is different. Dethloff (2002) suggests and shows that a solution algorithm designed for the VRPSPD may produce a good solution for the VRPBM. However, an algorithm designed for the VRPBM will not necessarily produce a solution for the VRPSPD because such an algorithm may not be able to handle VRPSPD's item traffic between the depot and the nodes.

This chapter's focus is limited to the VRPSPD case. In the *EBSCO Inspec* database using the key words "*vehicle routing*" and "*delivery and pick up*" together indicated nineteen records. Similarly, on *EBSCO Inspec* and *Ei Village* databases together, the combination of "*vehicle routing*" and "*backhaul*" yielded twenty four records. Non-relevant as well as out of scope articles were eliminated after the results were checked. The remaining ten studies constitute an exhaustive list of extant VRPSPD literature until 2007. Some other studies were detected that bear a similar or relevant name; however on further inspection, they were found to be instances of either the VRPM or the *dial-a-ride* problem rather than the VRPSPD.

3.2. A Taxonomy Model

The taxonomy presented at the previous chapter was the initial step for the rather more focused taxonomy presented in this chapter. For this study, the identified VRPSPD publications are mapped onto this previous taxonomy's second (Scenario Characteristics) and third (Problem Physical Characteristics) sections. The reason for excluding the other sections is that almost all identified articles appear in the same columns at those other sections. These studies have deterministic information without any dynamic or stochastic content. Synthetically generated data are accessible globally and processed centrally.

Mapping results show that many columns in the previous taxonomy are empty. Some of the less relevant or non-addressed categories and attributes, therefore, are either eliminated or combined. Since no recorded study has incorporated stochastic or dynamic data attributes that content has been removed. Those may easily be added as necessary in the future. At this point, only some minor additions to the already existing taxonomy were found to be necessary. The test article introduced by Salhi and Nagy (1999) incorporate an upper limit on maximum route lengths. Besides, they introduce multiple depot configurations with vehicles always returning to the depots from which their tour was initiated. Thus, besides adding the maximum tour length attribute, the previous VRP taxonomy's subcategories 3.4 (Number of points of origin) and 3.5 (Number of points of loading/unloading facilities (depots)) merge to become the new taxonomy's subcategory 1.3. The categories are also redefined along with their content. The new partitioning provides the possibility of deriving a practical representational scheme. Figure 3.1

provides the newly introduced taxonomy for the VRPSPD, and Table 3.1 provides the exhaustive list of VRPSPD articles mapped on this taxonomy.

1. Network and nodes
    1.1. Transportation network design
        1.1.1. Directed network
        1.1.2. Undirected network
    1.2. Geographical location of addresses
        1.2.1. Urban (scattered with a pattern)
        1.2.2. Rural (randomly scattered)
        1.2.3. Mixed
    1.3. Number of loading/unloading facilities (depot)
        1.3.1. Single depot
        1.3.2. Multiple depots

2. Vehicle attributes
    2.1. Number of vehicles
        2.1.1. Exactly *n* vehicles (*n > 1 o/w it is a TSP*)
        2.1.2. Up to *n* vehicles
        2.1.3. Unlimited number of vehicles
    2.2. Vehicle homogeneity (Capacity)
        2.2.1. Similar vehicles
        2.2.2. Heterogeneous vehicles
        2.2.3. Customer specific vehicles (A customer must be visited by a specific type of vehicle)
    2.3. Vehicle Route Length
        2.3.1. Unbounded
        2.3.2. Bounded

3. Service requirements/specifications
    3.1. Time window structure
        3.1.1. Soft time windows
        3.1.2. Strict time windows
        3.1.3. Mix of both
    3.2. Time window type
        3.2.1. Restriction on customers
        3.2.2. Restriction on roads
        3.2.3. Restriction on depot/hubs
        3.2.4. Restriction on drivers/vehicle
    3.3. On site service/waiting times
        3.3.1. Deterministic
        3.3.2. Time dependent
        3.3.3. Vehicle type dependent
    3.4. Time horizon
        3.4.1. Single period
        3.4.2. Multi period

Figure 3.1 Taxonomy of the VRPPD literature

4. Time structure and cost
    4.1. Travel time
        4.1.1. Deterministic
        4.1.2. Function dependent (a function of current time)
    4.2. Transportation cost
        4.2.1. Travel time dependent
        4.2.2. Distance dependent
        4.2.3. Vehicle dependent (Cost of operating a vehicle is not negligible)
        4.2.4. Operation dependent
        4.2.5. Function of lateness
        4.2.6. Implied hazard/risk related

Figure 3.1 (continued) Taxonomy of the VRPPD literature

As shown in Table 3.1, some empty columns still appear. In the first and second sections, the empty columns belong to those attributes that have not yet been invoked in the very specific VRPSPD literature, but these attributes are rather common for the VRP. The unused attributes of the third section are related to time issues. Although the VRP literature has recorded a large number of papers with a variety of time window constraints, such constraints have not been embedded into the VRPSPD literature except for the study by Angelelli and Mansini (2002). Thus, more VRPSPD instances with time window constraints are expected to appear in the future.

Table 3.1 Mapping of the research on the modified taxonomy framework for the VRPPD

Articles	1.	1.1.	1.1.1.	1.1.2.	1.2.	1.2.1.	1.2.2.	1.2.3.	1.3.	1.3.1.	1.3.2.	2.	2.1.	2.1.1.	2.1.2.	2.1.3.	2.2.	2.2.1.	2.2.2.	2.2.3.	2.3.	2.3.1.	2.3.2.
Min (89)				x		x				x				x				x				x	
Salhi and Nagy (99)				x		x	x	x		x	x					x		x					x
Dethloff (01)				x		x	x	x		x	x			x		x		x				x	x
Dethloff (02)				x		x	x	x		x	x			x		x		x				x	x
Montane and Galvao (02)				x				x		x						x		x				x	
Angelelli and Mansini (02)				x		x	x	x		x						x		x				x	
Nagy and Salhi (05)						x	x	x		x	x					x		x					x
Chen and Wu (06)				x		x	x	x		x	x					x		x				x	x
Montane and Galvao (06)				x		x	x	x		x	x			x		x		x				x	x
Bianchessi and Righini (07)				x		x		x		x						x		x				x	

Articles	3.	3.1.	3.1.1.	3.1.2.	3.1.3.	3.2.	3.2.1.	3.2.2.	3.2.3.	3.2.4.	3.3.	3.3.1.	3.3.2.	3.3.3.	3.4.	3.4.1.	3.4.2.	4.	4.1.	4.1.1.	4.1.2.	4.2.	4.2.1.	4.2.2.	4.2.3.	4.2.4.	4.2.5.	4.2.6.
Min (89)																x				x				x				
Salhi and Nagy (99)																x				x				x				
Dethloff (01)																x				x				x				
Dethloff (02)																x				x				x				
Montane and Galvao (02)																x				x				x				
Angelelli and Mansini (02)				x			x									x				x				x				
Nagy and Salhi (05)																x				x				x				
Chen and Wu (06)																x				x				x				
Montane and Galvao (06)																x				x				x				
Bianchessi and Righini (07)																x				x				x				

The taxonomy's fourth section in Figure 3.1 deals with cost structures and the ways they are represented. Currently the literature defines cost as the total distance covered by all vehicles of the fleet. However, as time issues and other constraints are added, cost computation based on total operational time and lateness may well be observed. Also a more complex vehicle mix configuration will add vehicle or operation specific cost modeling to the literature. Hazardous material (*HAZMAT*) transportation is a relatively new area that introduces different costing structures based on the implied risk on populations residing in proximity to roads. A simple way to calculate the implied risk is to multiply the partially or wholly covered distance with some deterministic risk factor. Then, the cost model reduces simply to being only distance dependent (4.2.2.) of Figure 1. However, Erkut *et al*. (2005) list many other complex models for risk calculation. A *HAZMAT* transportation cost modeling structure is expected to appear in the VRPSPD literature since some applications of this type problem already exist in the real world, and these include but are not limited to commercial distribution of combustible industrial gases in reusable cylinders.

3.3. Organization of the Literature

In this study, the previously identified VRPSPD papers are classified based on two schemes. The first scheme, introduced by Reisman (1988, 1992) and applied in Reisman and Kirschnick (1995), distinguishes among the different research strategies used. The second scheme (Silver, 2004) focuses on the studies' solution algorithms. These two schemes help identify the paper's importance and the contribution.

Reisman and Kirschnick (1995) identify “seven different, but not mutually exclusive or necessarily completely comprehensive” process categories of research strategies used in the OR/MS area. Brief definitions of each category follow (Gattoufi *et al*., 2004):

1) Ripple:An extension of previous theoretical or applied type of research in a given discipline or subdiscipline.

2) Embedding: Development of a more generalized formulation or a more global theory by embedding several known models and theories.

3) Transfer of technology: Using what is known in one discipline to model a problem domain falling in some other, perhaps disparate discipline.

4) Bridging: Combining known models or known theories that results in the growth of some initially unrelated fields of knowledge.

5) Creative application: The direct, not analogous, application of a known methodology to a problem or research question that was not previously so addressed.

6) Structuring: Organization and documentation process of phenomena not previously structured axiomatically or in model form.

7) Statistical modeling: Deriving models that arise from statistical manipulations and from analyses performed on empirically obtained data.

Table 3.2 maps the papers identified as dealing with the VRPSPD, according to the respective research strategies used by their authors. Although the convention has been to identify the primary and secondary process categories for each study (Reisman and

Kirschnick, 1995 and Gattoufi *et al*., 2004), because this set of papers is small, a single identification and a discussion of each paper was considered to suffice.

Table 3.2 Mapping of previous studies based on research category classification

Articles	**Ripple**	**Embedding**	**Transfer of Technology**	**Bridging**	**Creative Application**	**Structuring**	**Statistical**
Min (89)					X	X	
Salhi and Nagy (99)		X		X		X	
Dethloff (01)	X				X		
Dethloff (02)	X		X				
Montane and Galvao (02)			X				
Angelelli and Mansini (02)			X		X		
Nagy and Salhi (05)	X			X			
Montane and Galvao (06)	X		X				
Chen and Wu (06)	X		X				
Bianchessi and Righini (07)	X		X				

The last scheme (Silver, 2004), Table 3.3, provides an elaborate study and classification of solution algorithms that deal with complex OR/MS problems. These problems are not limited to combinatorial ones, but also comprise the non-linear, and stochastic versions. The comprehensive list of algorithms is grouped under two sections: basic types of heuristic methods and meta-heuristics. The list of basic heuristics and brief explanation for each one is given below. The heuristics are not mutually exclusive in application; i.e., a paper may incorporate more than one heuristic method, even a meta-heuristic, while generating solutions. Silver (2004) provides problem types, heuristic models, and references for each item in the list below.

1) Randomly generated solutions: Randomly generate feasible solutions and evaluate each one to determine the best.

2) Problem decomposition/partitioning: Partition the problems into smaller sub-problems and deal with each individually to generate one wholesome solution.

3) Inductive methods: Provide solutions by generalizing from smaller versions of the same or analogous problems.

4) Methods that reduce the solution space: These methods drastically cut back the number of solutions before they are even considered.

5) Approximation methods: Manipulate the mathematical model to simplify the original version of the model.

6) Constructive methods: Use the data of the problem to construct a solution step-by-step so no solution is generated until the process is complete.

7) Local improvement, or neighborhood search, methods: These improvement and search methods do not have mechanisms to allow exploring new solutions after reaching local optima.

These basic heuristics have one common characteristic: they find a solution and terminate. They lack the capability to explore the solution space without being stuck at local optima. Thus, improved heuristics with some level of guidance capability are introduced as meta-heuristics. These master processes guide and modify subordinate heuristics to reach high quality solutions efficiently. Besides basic heuristics, Silver (2004) lists nine main types of meta-heuristics and provides some instances of use, problems, and references. A list of those algorithms appears as follows:

1) Multilevel refinement

2) Beam search

3) Tabu search

4) Simulated annealing

5) Variable neighborhood search

6) Guided local search

7) Multi-start constructive approaches

8) Ant colony search

9) Evolutionary algorithms

All ten published VRPSPD papers are checked for the algorithms they use. In the case of meta-heuristics applications, some basic heuristics are also indicated because most meta-heuristics require a basic heuristic either to generate an initial solution or to improve the solution they created. Table 3.2 provides a classification of the identified literature based on the process category of the research strategy while Table 3.3 provides mapping of the presented algorithms on the solution methods scheme.

Table 3.3 Classification of the previous work based on the solution method used

	Basic Heuristic Methods							Metaheuristics								
Articles	Randomly Generated Solutions	Problem Decomposition	Inductive Methods	Reduce Solution Space	Approxomation Methods	Constructive Methods	Local Improvement	Multilevel Refinement	Beam Search	Tabu Search	Simulated Annealing	Variable Neighborhood Search	Guided Local Search	Multistart Constructive Approaches	Ant Colony Search	Evolutionary Algorithms
Min (89)		X			X								X			
Salhi and Nagy (99)		X				X										
Dethloff (01)						X								X		
Dethloff (02)						X								X		
Montane and Galvao (02)		X				X	X									
Angelelli and Mansini (02)		X		X	X							X				
Nagy and Salhi (05)		X				X	X									
Montane and Galvao (06)						X				X						
Chen and Wu (06)						X	X			X	X					
Bianchessi and Righini (07)						X	X					X				

3.4. Review of the Literature

The first study on the VRPSPD is attributed to Min (1989), who studied book delivery traffic among 22 libraries and a main library by using two vehicles of equal capacity, and he offered a two-stage solution method. The first stage grouped customers into clusters based on geographical proximity, and within each cluster, a TSP without load constraints is solved by using a branch and bound technique. Obtained routes are assessed, and any load infeasible sequences are broken by adding a penalty value to the arc distance. Afterwards, the relaxed TSP with penalty cost is solved again. The clustering phase exemplifies '*problem decomposition*,' and relaxing the load constraints of TSP reveals an instance of an '*approximation method*.' Changing the arcs' cost coefficient is an example of altering the weights of decision variables on the objective function; thus the search process may be called a '*guided local search*' procedure.

Because Min (1989) performed the first recorded study in this field, it shall be categorized as '*structuring*.' Besides, he developed the first solution method in this field, which puts this study in '*creative application*' category as well.

After a long and silent era, the second paper by Salhi and Nagy (1999) makes the first grouping and identification of different VRPDP types based on load structure. Mixed load (VRPBM) and simultaneous cases are distinctively identified and focused.

Salhi and Nagy (1999) solve both the VRPSPD and VRPBM problems. First, a VRP for linehaul customers is solved, and then backhaul customers are imbedded into tours in singles, doubles, or clusters by using three different algorithms. This approach first separates nodes into two sets and solves for one, representing a '*problem* *decomposition*' method, and imbedding nodes in a rather complex, greedy way then '*constructs*' solutions. They test their algorithms on self-derived problem instances. 70 of those instances have a single depot, and 55 of them have multi-depot configuration with numbers of depots ranging between two to five. They also introduce an upper limit on a possible route's maximum length. In multi-depot cases, all vehicles return back to the depot of origin. The test networks/systems introduced have been widely adopted in ensuing research by others, such as Dethloff (2001, 2002), Nagy and Salhi (2005), Chen and Wu (2006), and Montane and Galvao (2006).

3.5. Classification of Papers

Salhi and Nagy (1999) not only strucuture the research space by identifying different problem settings, but they also introduce the first practical benchmark data sets. They define different problem types that previously used to be named under a single title. Thus, their work may well be categorized as '*structuring*' and '*embedding*.' They also '*bridge*' the VRPBM and VRPSPD and further improve VRPBM algorithms in order to solve both. Thus, this study is marked as a '*bridging*' study.

Dethloff (2001) introduces a complex version of the cheapest insertion algorithm for VRPSPD. The algorithm starts with a seed node and constructs a simple back-and-forth route with the depot. Then, each node is assessed on the truck's residual capacity and the minimum possible addition to the total tour length. A function that blends these two picks the best node to insert at the best location on the current tour. The process iterates until the vehicle's capacity is exceeded. Sequentially, another tour starts, and this process is repeated until no node is left unassigned. This process repeats once for each node as the seed node and picks the best solution. Its classification will be discussed in conjunction with Dethloff's next paper.

Dethloff (2001) introduces 40 random VRPSPD instances besides those introduced by Salhi and Nagy (1999) and Min (1989). Dethloff (2002) further applies the same algorithm developed for VRPSPD on VRPBM instances specified by Salhi and Nagy (1999) and proves that his model outperforms the previous model which was particularly designed for VRPBM instances. Dethloff (2002) shows that VRPSPD algorithms are applicable to VRPBM instances; however, he warns that the reverse is not necessarily true. Dethloff (2001, 2002) uses the same algorithm in both studies, a '*constructive heuristic*.' Since the constructive process repeats for each node picked as the seed node, the algorithm can be identified as a '*multi-start meta-heuristic*.' These two studies utilize previous results as benchmarks. Thus, they classify as part of the '*ripple*' research category. Although together they could be taken as another '*bridging*' research between VRPSPD and VRPBM, the earlier is identified under '*creative application*' category while the latter one qualifies as part of the '*transfer of technology*' category.

Montane and Galvao (2002) define three different types of VRPDP, one of which adds to the previously defined three. The introduced problem type, which is referred to as the *Express Delivery Problem*, constructs separate delivery and pickup routes to serve customers who have either pick up, delivery, or both service type requirements. First, delivery routes are constructed and each vehicle handles delivery needs while traversing its route. Then, each vehicle starts its pick-up route that services a subset of nodes, which is not necessarily the same set of nodes visited during the delivery service. Thus, each node may be visited twice, possibly by two different vehicles, so as to satisfy the two types of service. Their algorithm for the VRPSPD starts with two well known heuristics: tour partitioning and sweep heuristics. The original problem is divided into TSPSPD sub-problems; and those are solved using cycle, minimum spanning tree, and cheapest insertion heuristics. Node exchange operators are used to overcome route infeasibilities and improve solution quality. The authors generated eight heuristics and tested them on 27 problems of their own creation. The number of nodes for these problems changes between 32 and 80. Min, max, and average solution values are provided rather than best results for each problem instance.

Hence, the Montane and Galvao (2002) work may be classified as a '*transfer of technology'* kind of research since the method followed combines and adapts currently existing methods to a relatively new problem. Partitioning the main problem into sub-problems qualifies as a '*problem decomposition*' method. '*Constructive methods*' such as the cycle heuristic, the cheapest insertion heuristic, and node exchange/swap kind of local improvement techniques are used to solve each of the sub-problems.

Angelelli and Mansini (2002) introduce window constraints to the VRPSPD. They generate 30 data sets by modifying the Solomon (1987) instances. Their research is the first and the only study to use an exact algorithm for solving a version of the original problem. They developed a branch and price algorithm in a set covering formulation. A relaxation of the elementary shortest path problem with time windows and capacity constraints is used as a pricing problem. A branch and bound technique is used to obtain integer solutions. Variants of both models are developed and applied to the problem. Inasmuch as the authors modified and improved currently available solution techniques to the problem makes this research a '*transfer of technology*.' However, because it is the first exact solution technique for this problem, the research also demonstrates a '*creative application*' characteristic as well. The study reduces the original problem to a set covering problem by applying partitioning techniques. Thus, it may be marked as a '*problem decomposition*' algorithm. Integer constraints are relaxed during the pricing phase of the solution, and they are added by applying a branch-and-bound technique so the algorithm benefits an '*approximation method*' and '*reduces the solution space*.' The main search algorithm, which is the '*price*' phase, alters the coefficients in the objective function; thus it constitutes a '*guided local search*' meta-heuristic.

Nagy and Salhi (2005) revisit their previous research to extend and update it. Once again, they deal with the VRPSD and with the VRPDP together when both problems have the same attributes. They provide a list of VRPDP articles with their respective main features, and they improve their previous (Salhi and Nagy, 1999) constructive heuristics by adding more node operators that lead to solution refinement.

They introduce three new heuristic methods and compare their performances to three other methods together with the best performing method of their previous study. This research is grouped under the '*bridging*' research category because it combines two problem types. Because they compare their results with the results of previous heuristics, the research is also categorized as '*ripple*.' Nagy and Salhi (2005) add some solution refinement methods and more complex node operators to their previous algorithms. Thus, together with the previous study's heuristic identification, this time the '*local improvement*' contribution is marked on the solution methods table.

Chen and Wu (2006) provide a study of the same problem. They develop two algorithms, and the first is an insertion based heuristic, which also provides the initial solution for the second algorithm. The second algorithm is a hybrid meta-heuristic which works like the '*Simulated Annealing'* method, but it replaces a deterministic rule for the probabilistic moves. The algorithm further employs a '*tabu*' mechanism to avoid recurrence of previously visited local optima and finally refines the solutions by using node swap and exchange operators. The algorithm runs as long as improvement is obtained within a specific number of trials. The algorithm is run on 14 Nagy and Salhi (2005) examples and ignores the upper limit on route length. The authors claim better results than Salhi and Nagy's (1999) results; but they provide no comparison either to the improved 2005 results or to those provided by Dethloff (2001). They further generate some problem situations by modifying Solomon's (1987) examples by using the Salhi and Nagy (1999) method. This work is a '*transfer of technology*' application and a '*ripple*' study. The first phase of the algorithm is an instance of a '*constructive method*,' and the second phase comprises all '*tabu search*,' '*simulated annealing*,' and '*local improvement*' algorithms.

The study presented by Montane and Galvao (2006) was inspired by crew transportation between mainland Brazil and the open sea oil platforms that use helicopters. Since the distance a helicopter may fly between two spots is restricted by factors such as fuel capacity, the original problem comes with an additional constraint on the maximum length of a move between nodes. Swapping people between platforms is avoided, which makes this problem a fine VRPSPD example drawn from a real life situation. Their study uses a modified version of sweep and tour partitioning heuristics. Based on the net change of the total load and maximum distance constraint, the tours are either filled with nodes or closed. For this phase, four different selection rules are devised. The initial solutions constitute an input to the tabu search mechanism. The tabu search stops either when no further improvement occurs or an upper limit on the number of iterations is reached. The authors test their problem on 87 instances, provided by Dethloff (2001), Salhi and Nagy (1999), Min (1989), and 18 newly modified Solomon (1987) and extended Solomon (Gehring and Homberger, 1999) instances from the literature. The Montane and Galvao (2006) work is a good example of a '*ripple*' study. However, because it is the first tabu search based algorithm applied to the VRPSPD their research also falls in the '*transfer of technology*' category, rather than in '*creative application*' since the tabu search mechanism is not tailored specifically for this problem. Yet, in Table 3.3, the paper is marked as a *constructive method* and a *tabu search meta-heuristic*. The paper is not marked as '*problem decomposition*' because after sub-routes are constructed, tabu search deals with all of the routes together rather than dealing with each one separately.

Bianchessi and Righini (2007) apply local-search and tabu search algorithms on self-created random instances for the VRPBM. They also apply their algorithms to Dethloff's (2001) examples and report their improvement over those of Dethloff's solutions. However, for the same examples, they do not compare their results to those provided by previous researchers. They first construct initial solutions by using four different node selection rules, based on tolerance to capacity violations and overall tour feasibility. They further apply local search procedures by using node exchanges on the different neighborhoods that they define. Although the search procedure applied does not perfectly match the variable neighborhood search technique as it is defined in Silver (2004), it is better to locate the search algorithm in '*variable neighborhood search*' column in Table 3. Consequently, the study by Bianchessi and Righini (2007) provides a good '*transfer of technology*' by applying the tabu search, a well known technique on this problem. Also, their '*constructive*' heuristic combined with the '*local improvement*' method gives good results. Although they do not provide a detailed benchmark analysis, the comparison of their averages to those of Dethloff's (2001) for the same problem instances qualifies this study as a '*ripple*' type of work.

3.6. Summary and Conclusion

In this chapter a taxonomy model for the VRPSPD is proposed and its usage on all articles that represent the current literature in the field is illustrated. This taxonomy proves to be robust, parsimonious, and descriptive, yet it provides expansion capability for the future. Table 3.1 provides the basic ideas, configurations, and constraints of a number of research articles with adequate detail in a confined space.

The two other classifications (Reisman and Kirscnick 1995, Silver 2004) tabulate the research strategy and the type of solution method incorporated to deal with the problem. The three tables provide invaluable information on what has been done in VRPSPD and delineate specifications for problems as yet untouched by researchers.

CHAPTER IV

A HYBRID GRASP-GA APPROACH TO THE CAPACITATED VEHICLE ROUTING PROBLEM WITH SIMULTANEOUS PICK-UP AND DELIVERIES

4.1. Introduction and Problem Statement

The problem in this study is classified as a VRPSPD with a single depot and unlimited number of vehicles. The basic assumptions of the problem are (1) the vehicle fleet is homogeneous (i.e. each vehicle has the same capacity), (2) although the amounts picked up and delivered at each node are not the same, the probability distribution that generates those values share the same parameters, (3) feeding a customer with anything picked up at a node other than the main depot is strictly avoided, and (4) the quantities to be delivered and picked up are fixed and known in advance. The objective is to minimize the total distance covered by the fleet during service without violating the vehicle capacities.

Applications of this problem are observed in containerized drinking water and industrial gas distribution in reusable cylinders. The courier services are reorganizing their operations around this model in large metropolitan business districts as each office can directly ship parcels the same time they receive a delivery.

The remainder of the chapter is organized as follows: in the following section, literature is addressed. Section three provides a formal definition to the problem and section four elaborates the solution algorithm. Upon providing numerical results and benchmark figures at the fourth section, finally, concluding remarks and future research issues are presented in section five.

4.2. Notation and Problem Description

The formal definition of the VRPSPD problem is as follows:

Instance: A graph $G = (V, E)$ with edge weights w_e for all $e \in E$ and vertex weights d_v and p_v for all $v \in V$, a distinguished node, i.e. depot-d, and variable k denoting the number of vehicles available and used, and a parameter C denoting uniform capacity of each of the trucks.

Objective: Find a partition of the nodes in $V \backslash \{d\}$ to $V_1, \ldots, V_k$ and a subset of edges $T_k \subseteq E$ forming k tours each containing node d and each node of V_i exactly once, so *that* $\sum_{e \in Tk} w_e$ is minimized without violating $\sum_{j \in Vh} d_j \leq C$, $\sum_{j \in Vh} p_j \leq C$ for $h \in \{1, \ldots, k\}$ and $p_{vt}^* + d_{vt}^* + p_v \leq C$ for $v \in V$, $t \in \{1, \ldots, k\}$, where p_{vt}^* denotes all the load picked up at some partition V_t prior to some definite node $v \in V_t$; and d_{vt}^* denotes all the load to be delivered at some partition V_t after some definite node $v \in V_t$.

The notion and the mathematical formulation of VRPSPD is as follows (Dethloff, 2001):

Sets:

J: Set of nodes

J_0: Set of nodes including the depot such that $J_0 = J \cup \{0\}$

V : Set of vehicles

Parameters:

C: Vehicle capacity

c_{ij}: Distance between nodes $i \in J_o, i \neq j, c_{ii} = M, i \in J, c_{00} = 0$

D_j: Delivery amount of customer $j \in J$ from the depot

n: Number of nodes, i.e., $n = |J_0|$

P_j : Pick-up amount from customer $j \in J$

M: Large number, e.g. $M = \max\left\{ \sum_{j \in J} (D_j + P_j), \sum_{i \in J_0} \sum_{j \in J_0, j \neq i} C_{ij} \right\}$

Decision Variables:

l'_v: Load of vehicle $v \in V$ when leaving the depot (which can be eliminated from the model)

l_j: Load of vehicle after having serviced customer $j \in J$

π_j: Variable used to prohibit sub-tours (which can be interpreted as position of node $j \in J$ in the route)

x_{ijv}: Binary variable indicating whether vehicle $v \in V$ travels directly from node $i \in J_0$ to node $j \in J_0$ ($x_{ijv} = 1$) or not ($x_{ijv} = 0$)

Model:

$$\text{Minimize } z = \sum_{i\in J_0}\sum_{j\in J_0}\sum_{v\in V} c_{ij} x_{ijv} \tag{1}$$

Subject to

$$\sum_{i\in J_0}\sum_{v\in V} x_{ijv} = 1 \qquad j\in J \tag{2}$$

$$\sum_{i\in J_0} x_{isv} = \sum_{j\in J_0} x_{sjv} \qquad s\in J,\ v\in V \tag{3}$$

$$l_v^{'} = \sum_{i\in J_0}\sum_{j\in J} D_j x_{ijv} \qquad v\in V \tag{4}$$

$$l_j \geq l_v^{'} - D_j + P_j - M(1 - x_{0jv}) \qquad j\in J,\ v\in V \tag{5}$$

$$l_j \geq l_i - D_j + P_j - M\left(1 - \sum_{v\in V} x_{ijv}\right) \qquad i\in J, j\in J,\ i \neq j \tag{6}$$

$$l_v^{'} \leq C \qquad v\in V \tag{7}$$

$$l_j \leq C \qquad j\in J \tag{8}$$

$$\pi_j \geq \pi_i + 1 - n\left(1 - \sum_{v\in V} x_{ijv}\right) \qquad i\in J, j\in J,\ i \neq j \tag{9}$$

$$\pi_j \geq 0 \qquad j\in J \tag{10}$$

$$x_{ijv} \in \{0, 1\} \qquad i\in J_0, j\in J_0, v\in V \tag{11}$$

Figure 4.1 Mathematical formulation of the VRPSPD model

In the model above, the objective function (1) aims to minimize the total travel distance. Constraint (2) assures servicing each node exactly once. Constraint (3) assures that if a vehicle arrives at a customer, then the same vehicle must also leave it. Initial vehicle loads are determined by constraint set (4), while the initial loads after serving the first customer are defined with constraint set (5). The constraint set (6) introduces limits for vehicle loads "*en route*." The constraints (7) and (8) ensure load amount of a truck stay under the capacity limits. Constraints (9) are sub-tour elimination constraints and (10) are the related non-negativity constraints.

The model presented in Figure 4.1 introduces neither an upper limit nor an exact figure for the number of the vehicles. There is no explicit lower limit either. However, the context of the problem instances and the capacity limits impose strict lower bounds for the number of vehicles. When it comes to upper bounds, there exists none for all instances except for one. A possible explanation is that an optimized route will have not necessarily the possible minimum, but some number close to the minimum number of vehicles in the final solution. The reason is that each time a new vehicle route is initiated, a travel distance figure related with the trip back and forth to the main depot needs to be added to the total cost figure. Minimizing the objective function, thus, imposes an implicit constraint on the number of vehicles, hence, keeps it at a possible low figure.

A relaxation of the VRPSPD may be obtained by separating pick-up and delivery processes such that at any node, either pick-up or a delivery occurs. This relaxation has been commented to be at least as hard as the *NP-hard* problems to solve (Mosheiov, 1998). Thus, the VRPSPD is also *NP-hard* in the strong sense.

4.3. The Hybrid GA Approach for the VRPSPD

Genetic Algorithms (GA) have recently been used to solve various routing problems. However, to the best of our knowledge, there is no GA developed to solve the VRPSPD. The solution method that is proposed here is referred to as a hybrid GA approach since it consists of three phases; a construction, genetic cross-over, and an improvement phase. These phases are described in detail in the following sub-sections. The VRPSPD requires an efficient data structure to handle the needs of the evolutionary mechanisms in a GA. Therefore, rather than the typical binary representation, integer labels denoting the customers are used to address the alleles of the genes. This avoids creation of meaningless genes which is a quite common phenomenon while using the original binary form of GA encodings. Using integers on genes reduce efforts for reiterating chromosomes to generate meaningful phenotypes. For a detailed overview of GAs see the following works by Gen and Cheng (2000) and Reeves and Rowe (2003).

A summary of the steps of the proposed solution algorithm is given in the Figure 4.2. The first step is to initiate the population. For this purpose a simple GRASP method is developed. This part is described as the construction phase. This phase is followed by the generations, which is the GA part. Before the crossovers, each chromosome is partitioned into tours and these tours are further improved using local search. Sum of all tour lengths constitute the fitness value of each chromosome. Based on this value, chromosomes and their mates are picked for cross-over.

Initiate the population by *GRASP* method
While Number of generations ≤ Criteria on Generations
  For each chromosome
    Partition chromosome into tours
    Improve tours by *local search* (1 node Or-opt)
    Evaluate the chromosomes and calculate fitness values
  End For
  If Number of generations without any improvement ≥ *threshold*
    Create new chromosomes using relaxed *GRASP* method
    Replace bad performing chromosomes with the new ones
  Else
    Replace bad chromosomes with good ones in the same population
  Perform *crossovers*
  Perform *mutations*
End While

Figure 4.2 Description of the hybrid GA algorithm

4.3.1. Construction Phase

Although there is no clear evidence or consensus on the hypothesis that "a better initial population leads to better solutions in shorter times", in this study on GA, the first phase of the solution algorithm is designed so as to generate a good quality initial population. Thus, the random keys method (Bean, 1994) that was initially used in (Vural, 2003) is replaced with a simple greedy randomized adaptive search procedure (GRASP), which is also a construction type heuristic.

Construction type heuristics aim to provide feasible as well as reasonably good solutions to complex problems. Lacomme *et al.* (2001) emphasize that most Genetic Algorithms (GA) for the TSP instances use permutation chromosomes as constructive heuristics. For the capacitated arc routing case that they study (which is a special type of TSP), a chromosome could be viewed as the order in which a vehicle must perform *n* tasks, assuming that a single vehicle performs all trips in turn. This encoding type is found appealing because it always offers a feasible sequence. However, that one great trip may be divided into sub-trips considering all the inevitable trip delimiters such as the capacity constraints. This type of tour partitioning process is called the "iterated tour partitioning procedure (ITP)."

Since its introduction (Feo and Resende 1989, 1995; Hart and Shogan, 1987) GRASP has found itself a great deal of applications in combinatorial optimization (Festa and Resende, 2002). GRASP can be applied either in sequence or in parallel. In the first, routes are constructed sequentially, i.e. a new route is initiated only after a current route is closed, and for the second, predefined number of routes are constructed simultaneously starting from a seed and consecutively adding new structural elements one at a time. In our solution procedure, a sequential GRASP is preferred since it provides ease in coding.

GRASP is a multi-start two phase procedure. The construction phase builds a feasible solution, whose neighborhood is investigated until a local minimum is found during the local search phase. The best overall solution is kept as the result (Resende and Ribeiro, 2003). This procedure repeats itself a predefined number of times and terminates given a specific condition of iterations.

At each iteration of the construction phase of GRASP, a list of candidate elements that can be incorporated into the partial solution is formed. Then, each of these candidates is evaluated using a greedy function, which in most cases is a function of increment in the objective function. A smaller list of candidate elements is formed (Restricted candidates list – RCL), in which the next element to be picked is selected by a random choice. A detailed review of this process is provided in (Resende and Ribeiro, 2003). The local optimization phase of GRASP is performed at the "route improvement phase" of the algorithm, which is described in the following sections.

In our algorithm, GRASP is used first to generate of the initial population. A sequential way of route construction is preferred over parallel method since this helps keep the number of routes as low as possible and cuts efforts to further combine routes with slack capacity. First, a node is picked randomly as the first seed. Then, a RCL is constructed among the closest nodes around the seed that also satisfy the capacity constraints. For each candidate, capacity is checked for both delivery (i.e. availability of items to meet the demand) and pick up (i.e. availability to accommodate the picked items following the delivery). RCL is limited with α (Alpha) number of nodes among which next element to be appended to the partial solution is chosen. In case there is less number of elements in RCL than α, an element is chosen randomly among the available candidates in RCL. If there is not a candidate available, then current route is closed and a new route is initiated by picking a new seed. Although the first seed is picked randomly, the following seeds are picked by a special rule, i.e. *triangularity constraint*, beside capacity constraints. This rule imposes that the angle between the depot and the lines that connect the previous and the candidate seeds must be an obtuse angle. This constraint assures that the seeds are selected as disperse as possible (not necessarily as far as). This type of scattering of the seeds assures that vehicle routes are not concentrated among the nodes close to the depot and follow the one previous seed, but those nodes far away from the main depot are also included evenly in the constructed routes. If there is no node available that can satisfy this triangularity constraint, a new node is picked randomly among the currently available ones that can satisfy the capacity limits. Figure 4.3 provides the basic steps of this mechanism.

```
Repeat for each chromosome
Pick randomly the first seed
While Number of appended nodes ≤ Total Number of nodes
        If the candidate is a seed
          CandidateNode ← first available node
          If angular and capacity constraints are satisfied
              NewNode← CandidateNode
          Else move to the next available node
        Else (candidate is not a seed)
          While Number of Nodes in RCL≤ α and there exists available nodes
              If the candidate satisfies capacity constraints
                  Add it to RCL
                  Move to the next available node that is closest to the NewNode
          End of While
          If RCL>0
                  Pick randomly the NewNode in RCL
          Else
                  NewNode will be a seed.
 End While
```

Figure 4.3 Description of the GRASP procedure

The same procedure above is relaxed and used later in the program to create new chromosomes to add into the current population in case no improvement is realized for a large number of generations. For the relaxed case, available pick-up capacity check for free capacity is ignored. Also the angular constraint for new seed selection is traded for a random selection of seeds among the available nodes. The reasons for adding new chromosomes into the population after some number of generations are (1) avoiding the convergence of the population to one "best" chromosome after a good number of generations and (2) adding new gene permutations to the population so that a better sequence may be generated by blending relatively poor but "fresh" chromosomes with improved but "old" ones. The number of "fresh" chromosomes is limited at some low value initially; and reaches one half of the population at max before being reset to zero. The reason of keeping this value low is to slowdown or avoid the converging process of the population to a local optimum. And the reason for renewing half of the population while keeping the other half is to maintain characteristics of relatively good quality solutions in the population. This approach provides a control on the convergence of the population to a local optima and ability to pursue the search in the solution space without being stuck at a local optimum.

4.3.2. Genetic Cross-over Phase

One important need in GAs is maintaining the feasibility of a sequence throughout the crossover operations. In many cases, after the reproduction phase, one may see replication of genes in one parent chromosome while the other parent lacks those in its sequence. Missing a gene in the sequence means not servicing that node while a replication means serving it twice. An efficient method should avoid both lacking and replicating a gene, which add to the computational complexity and solution time. After reviewing quite a number of studies for a cross-over strategy that will minimize computational times, two studies are identified to satisfy our needs. Among those two, the random keys method by Bean (1994) is shown (Vural, 2003) to generate inferior results over the method proposed by Topcuoglu and Sevilmis (2002). So, in this study, the latter mentioned method is picked for the cross-over mechanism.

Topcuoglu and Sevilmis (2002) develop a GA model for a task-scheduling problem with multiple objectives. They define a chromosome as a single string with a predefined length. The string represents the number of tasks to be scheduled. The ordering of tasks on the chromosome shows the precedence relations of tasks on the processors. Their single-point crossover operator randomly generates a crossover point and cuts the selected pair of parent chromosomes into left and right parts and creates two offspring chromosomes. One offspring is created by taking the left part of the chromosome from one of the parents and copying the missing genes (the ones in the right part of the chromosome) from the other parent. It is important to copy the missing genes in the order they appear in the other chromosome. The other offspring is produced similarly. This strategy guarantees the production of valid sequences without missing or duplicating any genes. Passing blocks of genes to the offspring enables inheriting genetic information to new generations.

The GA crossover operation in this study performs just as described above, however tasks are not coupled with assigned processors in our chromosome strings. Genes comprise only a number which denotes a specific node. Figure 4.4 illustrates the process that is described at the previous paragraph. At each generation, chromosomes are partitioned into tours based on capacity constraints and tours are sent to the route improvement phase. Calculated lengths of the improved tours are summed to asses the fitness value of each chromosome. Based on these values, chromosomes are sorted in ascending order. Following the sorting, mutations and necessary replacements for bad chromosomes are performed. Replacements are two types: (1) replacing a given number of bad chromosomes with good ones or (2) replacing those with increasing number of newly generated chromosomes using relaxed GRASP, which is described before. Replacement process starts after a certain amount of generations pass without any improvement in the current best total tour length and number of replacements increase as this value surpass certain threshold values. Maximum number of replacements is a quarter of the population, which is immediately followed by resetting the count of generations without any improvement to a much lower value. Mutation is performed as swapping the location of two nodes in the same chromosome.

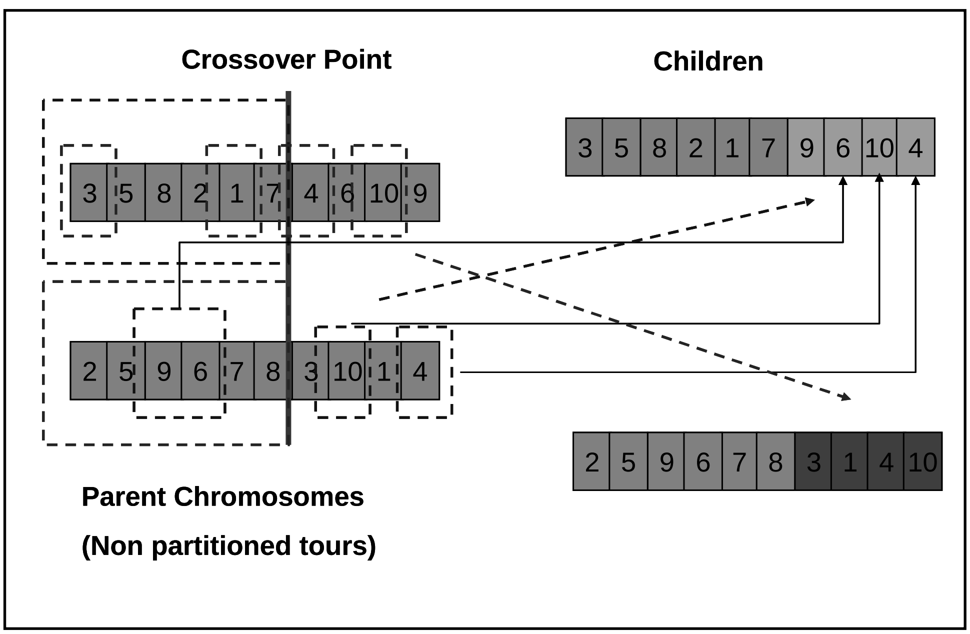

Figure 4.4 The cross-over process

The chromosomes are grouped under three segments, top 25 percentile, following 25 percentile, and the remaining half based on their fitness. Top 25 percentile is given two probability points and the others are assigned only one. Having three partitions increases the chance of a parent selected from the good performing top half, which is given three probability points compared to the lower half which is given only one. Further, the probability of a selection among the top quarter is doubled compared to the chance of the following second quartile. This is assured by assigning two probability points to the top 25 percentile, while assigning only one to the ensuing 25 percentile. This pointing method always favors picking a parent among a good performing group.

A tournament selection is performed among these three segments. The first available chromosome among the chosen segment is picked as parent. If there is no available chromosome in this segment, then the first available in the following segment is picked for cross-over. After the crossover is performed among all chromosomes, another generation is initiated. Generations continue till a fixed number of them are performed.

4.3.3. Route Improvement Phase

Route improvement heuristics strive for constructing a better tour starting from an inferior performing one. The improvement phase of the algorithm reduces the length of tours using a local search method. In this phase, as soon as a vehicle route is established, each node is considered as candidate for replacement in a location between its current location and the tour end. The process starts with the first node of the tour by checking the difference between the amount to be delivered and amount to be picked. If this value is negative, it means that this node eats up free space that currently exists on the vehicle, thus can be moved to a further location. The replacement is realized at a location that provides the maximum savings on total tour length, if there exists any. If the net value of the load difference is positive, that means this node opens free space on the vehicle. However, this free space may be consumed by a following node, so shifting the location of this node to any succeeding location is not possible without any capacity violations. Rather, only shifting of such a node after the following node is checked for feasibility and gain in tour length. After relocation of the node is over, the same process is performed for the following node, until all nodes are iterated in the tour. Thus, our local search mechanism resembles the Or-opt (1976) process with a single node. Figure 4.5 illustrates the mechanism of this stage on a chromosome segment, i.e. route. For a more extensive survey on similar problem types and solution methods see Vural (2003).

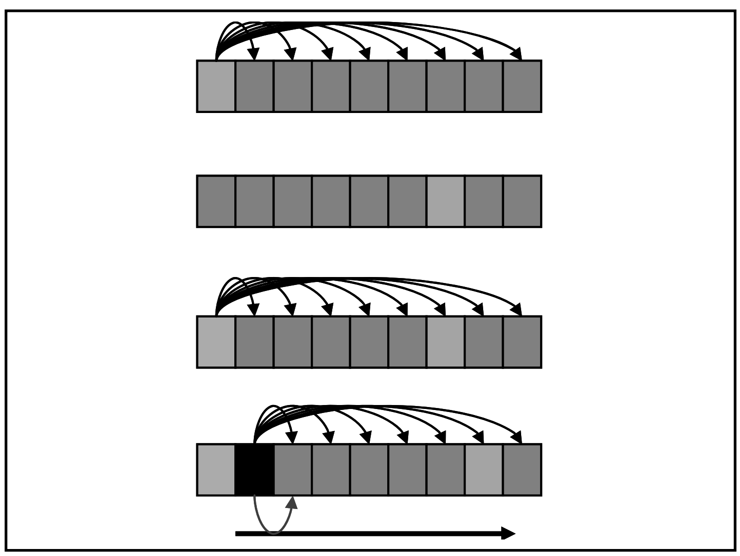

Figure 4.5 The route improvement process

4.4. Computational Study

The proposed hybrid GA is coded in ANSI C and tested on a set of 40 benchmark problems by Dethloff (2001) and 14 others provided by Salhi and Nagy (1989). Dethloff generates the test instances in a way to reflect urban and rural characteristics. In scenario SCA, the coordinates of customers are uniformly distributed over the interval [0, 100]. This is a random scattering inside a square and imitates rural landscape. Scenario CON concentrates one half the customers inside the interval [100/3, 200/3] which yields a more 'urban' settlement characteristic. Number of customers is kept at 50 and pick up quantity for each customer is generated using a uniform distribution over the interval [0,100]. Demand at any node j is calculated as $P_j = (0.5 + r_j)D_j$ such that r_j is uniformly distributed over the interval [0, 1]. The digit after the geographical scenario letter-triplets represents the minimum desired number of trucks. Dividing the total demand by this number yields the uniform truck capacity. By using two different minimum numbers of trucks and two different geographical scattering pattern, 4 different scenarios, and for each scenario, 10 problem instances are randomly generated.

The sample instances of Salhi and Nagy (2001) are generated by modifying the problems given by Christofides et.al. (1979). Those problems of Christofides were actually created as VRP instances. The demand quantity provided in the original problem context is distributed as demand and pick up quantities. For each customer node a, a ratio r_a is calculated as min((x_a / y_a), (y_a / x_a)), where x_a and y_a are coordinates of the customers. The new demand value of customer a is $d(a) = r_a .t(a)$ where $t(a)$ is the original demand value provided at the problem instance. The pick up quantity is simply calculated as $p(a) = t(a) - d(a)$. Another set of supply and demand levels was created by exchanging the demand and supply figures of every other customer. The former case is represented by letter *X* and the latter one is represented by letter *Y*. This way, 28 problems are created using the original 14, however due to presence of a limit on length of routes; we will use the 14 instances which are generated without this constraint.

The hybrid GA, like most other GAs, requires a large number of parameters that need tuning. On the other hand, robustness requires applicability of a solution procedure to similar problems with relatively a few numbers of parameters. After some computational tests, the number of parameters is kept at five. These parameters are:

- *Discard Quantity (m)*: the number of bad chromosomes (the worst *m*) that will be discarded with better ones (the best *m*) at each generation. Also fraction and multiples of this parameter is used to determine the number of chromosomes to be replaced with newly generated ones.

- *Initial Destructive Mutation Quantity:* the minimum number of chromosomes to be mutated (the maximum number is determined dynamically).

- *Population Size:* the number of parents that go under reproduction (the size of the population is kept constant throughout the generations).

- *Alpha (α):* The maximum size of the *RCL.*

- *FILLRATE:* Each truck needs to start its tour with some idle capacity unfilled. This provides the trucks to accommodate nodes with pick-up quantity larger than the demand quantity as the first seed. Although the free space on truck has a tendency to increase as the tour advances, this value may fluctuate, imposing some constraints sometimes, and lack of free space at the start of a tour makes any solution alternatives (permutations) with larger pick-up quantity infeasible.

The results for the Dethloff's set of benchmark problems are summarized in Table 4.1. The first column shows the problem name and the second column gives Dethloff's solutions (2001). The third column presents the best solutions found by Vural (2003) using a fixed parameter set. The results at the fourth column are generated by Montane and Galvao (2006) using a *tabu search* based meta-heuristic. They further provide the lower bound values for these problems which are on the fifth column. The sixth column is the results generated using our *hybrid GA* metaheuristic. The following two columns provide the percentage gap between our results and the best results in Montane and Galvao (2006), and the lower bounds respectively. The first percent gap is calculated as (*Column 4 value – Column 6 value*)/ *Column 4 value*. The second gap value compares the solution method's best to the lower bound. The calculation method is the same again: (*Column 4 value – Column 7 value*)/ *Column 4 value*. Table 4.2 provides a comparison of average values of 10 problems for each of the four different scenarios. Table 4.3 provides four different set of outcomes and the lower bound values beside our results for Salhi and Nagy (1999) instances. As seen, the proposed algorithm provides rather close results to best known values, and even beats one in 14 instances. However, the difference between the best known and generated using algorithm widens as the number of nodes increase. Appendix A provides the routes and minimum route figures for the best solutions obtained for the Dethloff (2001) instances. For each solution, first the problem number, number of generations required to generate the solution, and the cost figures are provided on the top line. Following this, vehicles tours are provided as clusters of nodes, which are lined up in the sequence they will be visited on the tour. Just like that, Appendix B provides the best results obtained for the Salhi and Nagy instances.

One major improvement of this study is on the solution times. The current algorithm produces solution values between 40 to 300 seconds based on problem attributes. The CPU seconds required to solve Dethloff instances can be calculated by dividing the number of generations reported by 100. As the number of nodes increase, time required to perform 100 generations increase from almost 1 second to 11 seconds observed for 200 nodes on a PC with 512 MB RAM and 2.6 GHz Intel Celeron (Mobile) CPU. So, for 50 nodes, generating a solution takes around 1 minute on the average. This value is a fraction of solution times reported by Volkan (2003) which are given to be around 40 mins on a PC with 1.8 GHz Intel Pentium 3 CPU and 256 MB RAM.

Table 4.1 Results of Hybrid GA and other methods on Dethloff instances

Problem Instance	Dethloff's Best	Vural's Best with Set Parameter	Tang and Galvao	Lower Bound	Results with Final Parameters	% Gap*	% Gap**
SCA3-0	689.00	680.25	640.55	583.77	647.31	1.06	10.88
SCA3-1	765.60	773.18	697.84	655.63	729.69	4.56	11.30
SCA3-2	742.80	722.54	659.34	627.12	703.94	6.76	12.25
SCA3-3	737.20	683.95	680.04	633.56	704.34	3.57	11.17
SCA3-4	747.10	736.33	690.50	642.89	709.57	2.76	10.37
SCA3-5	784.40	728.34	659.90	603.06	689.39	4.47	14.32
SCA3-6	720.40	670.06	653.81	607.53	687.48	5.15	13.16
SCA3-7	707.90	718.62	659.17	616.40	694.49	5.36	12.67
SCA3-8	807.20	758.09	719.47	668.04	754.37	4.85	12.92
SCA3-9	764.10	744.68	681.00	619.03	699.20	2.67	12.95
SCA8-0	1132.90	1129.25	981.47	877.55	999.31	1.82	13.87
SCA8-1	1150.90	1232.22	1077.44	954.29	1150.79	6.81	20.59
SCA8-2	1100.80	1232.62	1050.98	950.74	1187.86	13.02	24.94
SCA8-3	1115.60	1133.83	983.34	905.29	1094.45	11.30	20.89
SCA8-4	1235.40	1249.19	1073.46	972.62	1146.96	6.85	17.92
SCA8-5	1231.60	1157.54	1047.24	940.60	1081.70	3.29	15.00
SCA8-6	1062.50	1119.12	995.59	885.34	1095.80	10.07	23.77
SCA8-7	1217.40	1134.23	1068.56	955.86	1129.96	5.75	18.21
SCA8-8	1231.60	1192.88	1080.58	986.52	1130.33	4.60	14.58
SCA8-9	1185.60	1259.93	1084.80	978.90	1170.38	7.89	19.56
CON3-0	672.40	648.81	631.39	592.38	651.25	3.15	9.94
CON3-1	570.60	570.75	554.47	532.55	587.23	5.91	10.27
CON3-2	534.80	531.82	522.46	491.04	529.70	1.39	7.87
CON3-3	656.90	640.14	591.19	557.99	623.36	5.44	11.72
CON3-4	640.20	610.17	591.12	558.26	602.82	1.98	7.98
CON3-5	604.70	605.36	563.70	531.33	575.57	2.11	8.33
CON3-6	521.30	521.07	506.19	475.33	524.23	3.56	10.29
CON3-7	602.80	626.70	577.68	550.73	596.92	3.33	8.39
CON3-8	556.20	553.62	523.05	492.69	546.17	4.42	10.85
CON3-9	612.80	606.94	580.05	547.31	599.97	3.43	9.62
CON8-0	967.30	926.14	860.48	795.45	907.97	5.52	14.15
CON8-1	828.70	862.00	740.85	693.22	808.34	9.11	16.61
CON8-2	770.20	783.81	723.32	650.81	758.96	4.93	16.62
CON8-3	906.70	934.19	811.23	754.41	908.73	12.02	20.46
CON8-4	876.80	849.79	772.25	729.09	812.13	5.16	11.39
CON8-5	866.90	881.47	756.91	709.76	820.94	8.46	15.66
CON8-6	749.10	776.08	678.92	631.41	725.23	6.82	14.86
CON8-7	929.80	929.37	814.50	762.03	871.66	7.02	14.39
CON8-8	833.10	828.57	775.59	705.08	810.82	4.54	15.00
CON8-9	877.30	901.24	809.00	729.10	860.70	6.39	18.05
					Minimum	1.06	7.87
					Average	5.43	14.09
					Maximum	13.02	24.94

Table 4.2 Averages reported on Dethloff instances

Problem Group	Dethloff	Vural	Tang and Galvao	Bianchessi and Righini	Lower Bound	Results with Final Parameters
SCA3X	746.57	721.60	674.16	684.60	625.70	701.98
SCA8X	1166.43	1184.08	1044.346	1035.70	940.771	1118.75
CON3X	597.27	591.54	564.13	568.50	532.96	583.72
CON8X	860.59	867.27	774.31	776.40	716.04	828.55

Table 4.3 Results reported on Salhy and Nagy instances.

Problem Instance	No of Customers	Vehicle Capacity	Nagy and Salhi	Dethloff	Chen and Wu	Tang and Galvao	Lower Bound	Results with Final Parameters
CMT1X	50	160	525	501	478.59	**472**	454.68	484.22
CMT1Y	50	160	525	501	480.78	**470**	455.52	601.56
CMT2X	75	140	841	782	**688.51**	695	617.01	757.90
CMT2Y	75	140	839	782	**679.44**	700	617.64	725.78
CMT3X	100	200	829	847	744.77	**721**	646.73	774.74
CMT3Y	100	200	829	847	723.88	**719**	648.04	823.97
CMT4X	150	200	1053	1050	887.00	**880**	714.18	1103.39
CMT4Y	150	200	1047	1050	**852.35**	878	715.67	1247.09
CMT5X	199	200	1334	1348	**1089.22**	1098	858.14	1237.58
CMT5Y	199	200	1334	1348	**1084.27**	1083	856.59	1335.47
CMT11X	120	200	1087	959	**858.57**	900	663.38	1152.05
CMT11Y	120	200	1075	1070	**859.77**	910	662.84	925.87
CMT12X	100	200	820	804	678.46	675	568.79	802.56
CMT12Y	100	200	825	825	676.23	689	573.53	**664.42**

An important point to be highlighted at this study is that improving the quality of the initial population enhances the solution quality and reduces the process times for GA applications. As asserted before, the probabilistic population generation method of Bean (1994) is replaced with a more greedy selective process of GRASP. Although the generated results in limited time are not strictly better than those provided by *tabu search* powered methods, the hybrid meta-heuristic prove to be compatible with them as well.

4.5. Summary and Conclusion

The problem studied in this chapter is a special case of the classical VRP which is referred to as the VRPSPD. VRPSPD differs from the rest of the VRP literature mainly due to its unique capacity constraints. Besides, in this specific problem the tours are not segmented into line-haul and backhaul clusters and splitting of demand and pick-up quantities is not allowed. The proposed hybrid GA heuristic for the VRPSPD is tested on a set of benchmark problems and it generated relatively solutions with a considerable quality for most of the problems. The results show that hybridizing the GA mechanism with a greedy construction heuristic such as canonical GRASP enhances performance of the outcome. Although the improvement in the solution quality is significant compared to a previous GA study, yet the results are behind those generated by tabu search. The main reason is believed to be the high number of parameters that need to be tuned before running the algorithm and the considerable volume of random numbers employed in genetic algorithms. Future work may be directed towards reducing the number of parameters that need this kind of tuning or eliminating them if possible to produce even better results in confined amount of processor seconds.

CHAPTER V

CONCLUSION

In this dissertation, a very specialized case of the classical VRP, which is the VRPSPD, is studied. This problem differs mainly from the rest of the literature with its capacity constraints. These require simultaneous pick-ups and deliveries of loads of the same size from the depot to the customers and from the customers to the depot. Besides, in the case of the problem the tour is not segmented to line haul and backhaul clusters and splitting of the demand as well as pick-up quantities is not permitted to occur. Yet, there exists some number of similar instances to our problem in the literature. In this report these problems, approaches, and solution techniques in the literature as well as the general VRP literature are introduced and discussed.

The hybrid GA heuristic, which was developed by the researcher himself, start at one point of the search space and proceed down one path, accepting an improvement if it exists. Stochastic heuristics or heuristics that tend to change the search path based on upon probabilistic factors, yield to give different solutions during different executions, even when starting from the same initial solution (Thangiah and Petrovic, 1998). On the other hand, heuristics based on deterministic rules and policies tend to produce the same final solution no matter how many times one runs the heuristics with the same initial solution. The heuristics developed in this thesis incurred high level of stochastic content, not only due to the constructive GRASP mechanisms, but also due to the cross-over and mutational mechanisms of genetic algorithm. Thus, it is quite probable that even the runs are replicated with our codes on identical machines, one may not come up with the solutions same as the ones we have supplied at this work.

It was the first time at this paper a combination of GRASP and Genetic Algorithms were tested on a VRP instance. As a result of the study, some improvement figures are presented over the currently existing data. However, computation times, although not expressed explicitly, are not quite short to compete with other heuristics whose progresses are declared to be in seconds. Yet, the algorithm presented in this study still possesses practical applicability since the run times are not excessively large. Future directions to this work may be to eliminate the remaining parameters while improving the best solutions of this study as well as to enhance the speed of the process leading to a satisficing outcome. This would be probable by imbedding the improvements in the GRASP phase, improving the quality of the code in terms of run time, and enhancing the search capabilities of the genetic mechanism with those of the well performing ones such as the tabu search and the simulated annealing heuristics.

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APPENDIX A

BEST ROUTES FOR DETHLOFF (2001) INSTANCES

Problem: **SCA 530** Generation Optimum Reached: 4784 Solution: 647.31

Route 1: 1 26 20 50 11 47 5 44 16 24 15 32 37 25 38 22 35
Route 2: 41 7 2 40 30 14 39 9 4 12 17 27 8 10 36 48 34 29
Route 3: 43 28 42 49 19 46 31 23 3 45 18
Route 4: 33 21 6 13

Problem: **SCA 531** Generation Optimum Reached: 3610 Solution: 729.69

Route 1: 28 19 10 40 37 25 1 18 21 23 8 35 38 4 48 26 13 41 33 12
Route 2: 16 36 42 29 45 15 46 27 5 47 20 32 44
Route 3: 50 17 39 49 9 3 11 14 43 7 6 30 2
Route 4: 31 24 22 34

Problem: **SCA 532** Generation Optimum Reached: 5297 Solution: 703.94

Route 1: 17 3 40 42 11 21 5 34 7 15 49 30 48 18 23
Route 2: 46 26 41 16 37 1 35 50 44 39 4 6 13 20 29 2
Route 3: 14 45 8 31 19 27 32 12 10 25 9 47 43
Route 4: 33 36 38 24 22 28

Problem: **SCA 533** Generation Optimum Reached: 9364 Solution: 704.34

Route 1: 16 7 31 35 21 42 19 17 9 11 25 10 27 50 23 18 1 6
Route 2: 3 20 32 15 39 26 37 2 34 12 41 8
Route 3: 24 30 5 14 43 45 22 47
Route 4: 33 40 48 38 29 13 44 49 28 36 4 46

Problem: **SCA 534** Generation Optimum Reached: 7369 Solution: 709.57

Route 1: 23 24 11 22 36 29 6 47 5 49 43 7 4 27 2 41
Route 2: 9 37 34 15 32 17 3 14 50 38 35 12 19 25
Route 3: 21 46 42 40 18 48 16 31 33
Route 4: 44 26 1 28 13 39 8 45 20 30 10

Problem: **SCA 535** Generation Optimum Reached: 8375 Solution: 634.02

Route 1: 22 38 9 26 47 19 23 44 40 39 8 46 3 27 36 21 18 2
Route 2: 10 4 15 48 31 30 11 17 14 1 13 28 33 24
Route 3: 20 50 35 49 29 45 25 5 16 34 12 41 42 37
Route 4: 6 32 43 7

Problem: **SCA 536** Generation Optimum Reached: 7149 Solution: 687.48

Route 1: 17 37 46 11 7 4 16 42 40 38 25 43 33 9 30 19 28 50 35

Route 2: 26 39 23 2 6 32 20 13 21 34 5 45 8
Route 3: 49 24 15 44 48 10 18 47 12 1 3 27 41 29 31
Route 4: 22 36 14

Problem: **SCA 537** Generation Optimum Reached: 2760 Solution: 694.49

Route 1: 20 50 23 10 39 35 21 33 31 1 46 9 36 6 29 40
Route 2: 45 24 13 38 3 4 14 12 48 41 7 19 34 25 11 32
Route 3: 26 17 43 49 2 42 8 27 22 37 30 44 15
Route 4: 18 16 47 5 28

Problem: **SCA 538** Generation Optimum Reached: 4730 Solution: 754.37

Route 1: 12 48 44 3 43 20 32 34 30 16 40 10 41
Route 2: 23 49 26 39 1 17 36 38 42 7 47 2 4 37 25 33 24
Route 3: 5 13 45 19 8 31 50 6 46 21 18 35 22 29 11 15
Route 4: 28 9 14 27

Problem: **SCA 539** Generation Optimum Reached: 5045 Solution: 699.20

Route 1: 26 28 43 27 30 36 19 11 34 48 1 50 38 37 8 4
Route 2: 44 29 32 12 9 40 41 13 10 17 25 21 2 18
Route 3: 39 24 46 15 23 22 5 14 3 16 47 42
Route 4: 33 7 6 35 45 20 49 31

Problem: **SCA 580** Generation Optimum Reached: 8036 Solution: 999.31

Route 1: 40 2 7 41 18
Route 2: 29 34 48 36 10 25
Route 3: 8 27 17 12 4 9 39 14 30
Route 4: 35 22 24 16 44 20 1
Route 5: 38 15 32 37
Route 6: 33 3 45
Route 7: 43 21 6
Route 8: 13 26
Route 9: 28 19 49 42
Route 10: 50 11 47 5 23 31 46

Problem: **SCA 581** Generation Optimum Reached: 10834 Solution: 1150.79

Route 1: 40 25 18 1 2 50
Route 2: 44
Route 3: 31 49 9 11 14 3
Route 4: 7 43 6 30 37
Route 5: 42 45 15

Route 6: 27 46 5 47 20 32
Route 7: 36 16 29 48 26 33 12
Route 8: 21 23 8 35 38 4 13 41
Route 9: 10 19 28 34
Route 10: 22 17 39 24

Problem: **SCA 582** Generation Optimum Reached: 28451 Solution: 1187.86

Route 1: 23 48 49 15 7 21
Route 2: 11 42 40 34 5 18
Route 3: 32 12 10 25 38
Route 4: 35 37 16 41 26 46
Route 5: 45 31 19 14
Route 6: 2 28 24 22 29
Route 7: 36 9 47 43 27 8
Route 8: 17 3 1 39 6
Route 9: 20 13 4 44 50
Route 10: 30
Route 11: 33

Problem: **SCA 583** Generation Optimum Reached: 28518 Solution: 1094.45

Route 1: 24 34 2 37 26 12
Route 2: 6 25 10 27 50 23 18
Route 3: 48 29 13 44 46
Route 4: 47 22 45 5 14 1 43
Route 5: 4 49 28 36
Route 6: 19 17 11 35 9 42 16
Route 7: 39 41 30
Route 8: 8 32 15 38 40 33
Route 9:20 3
Route 10: 7 31 21

Problem: **SCA 584** Generation Optimum Reached: 5977 Solution: 1146.96

Route 1: 46 42 40 18
Route 2: 37 34 23
Route 3: 59 7 43 49 5 47 29 22
Route 4: 1 39 8 45 20 30
Route 5: 26 44 33 25
Route 6: 31 28 13 48 16
Route 7: 21 35 38 14 3 50
Route 8: 9 12 19 11
Route 9: 17 32 15 6 24
Route 10: 4 27 2 41 10

Problem: **SCA 585** Generation Optimum Reached: 7280 Solution: 1081.70

Route 1: 22 13 38 9 26 47 19 23
Route 2: 44 40 39 8 46 3 37
Route 3: 49 45 25 5 34 41
Route 4: 18 21 36 27
Route 5: 2 29 50 51
Route 6: 6 42 12 16 32
Route 7: 4 15 48 43
Route 8: 20 7 10
Route 9: 31 30 11 17 14 1
Route 10: 28 33 24

Problem: **SCA 586** Generation Optimum Reached: 38259 Solution: 1059.66

Route 1: 1 47 12 44
Route 2: 39 13 6 2 23 35
Route 3: 15 24 3 27 36 14 31
Route 4: 22 29 41 49
Route 5: 21 10 18 48
Route 6: 8 45 5 34 26
Route 7: 50
Route 8: 4 16 42 40 38
Route 9: 37 7 11 25 46 17
Route 10: 20 32 30 9 33 43 19 28

Problem: **SCA 587** Generation Optimum Reached: 3418 Solution: 1129.96

Route 1: 27 22 37 30 44 38 24
Route 2: 50 39 35 21 33 31 46
Route 3: 29 1 9 36 6
Route 4: 43 49 2 42
Route 5: 45 26 17 23 10
Route 6: 48 41 7 19 28
Route 7: 8 15 13
Route 8: 40 20 18 16
Route 9: 32 11 25 47 5
Route 10: 3 4 12 14 34

Problem: **SCA 588** Generation Optimum Reached: 12088 Solution: 1130.33

Route 1: 5 45 8 31 6 50 13 27
Route 2: 16 30 34 32 20
Route 3: 23 48 44 3 43
Route 4: 49 26 39 1 17 36 42 38

Route 5: 18 21 46 35
Route 6: 12 40 10 19 41
Route 7: 33 2 4 47 7 24
Route 8: 15 14 9 11
Route 9: 28 25 37 29 22

Problem: **SCA 589** Generation Optimum Reached: 7369 Solution: 1170.38

Route 1: 39 24 2 17 25 10 18
Route 2: 12 9 40 41 13
Route 3: 29 34 48 38 37
Route 4: 47 16 3 14 22
Route 5: 21 46 15 5 23 31
Route 6: 44 1 50 32
Route 7: 30 36 19 11 8 4
Route 8: 26 28 27 43
Route 9: 6 35 45 49 42
Route 10: 20 7 33

Problem: **CON 530** Generation Optimum Reached: 9767 Solution: 651.25

Route 1: 17 38 42 43 27 3 11 7 19 2 31 4 12
Route 2: 33 34 46 26 49 10 13 28 5 36 45 14 24 30 41
Route 3: 9 22 32 50 1 21 48 18 23 8 39 40 29 6 47 44
Route 4: 25 20 16 37 15 35

Problem: **CON 531** Generation Optimum Reached: 6774 Solution: 587.23

Route 1: 22 33 6 48 40 44 41 5 11 36 31 34 38 13 24 27 29
Route 2: 2 17 45 12 47 21 4 19 46 10 25 30
Route 3: 26 14 37 42 32 15 8 20 1
Route 4: 16 3 9 43 49 7 50 35 28 18 23 39

Problem: **CON 532** Generation Optimum Reached: 8259 Solution: 529.70

Route 1: 13 26 45 20 18 14 44 50 32 4 16 49 15 23
Route 2: 21 39 10 29 40 5 11 9 34 47 41 33 30
Route 3: 31 3 43 7 12 36 2 28 24 6 37 22 46 42 8 35 48 1
Route 4: 17 19 38 27 25

Problem: **CON 533** Generation Optimum Reached: 12130 Solution: 623.36

Route 1: 25 39 49 17 8 27 2 43 45 18 33
Route 2: 5 11 40 41 29 22 12 24 30 38 6
Route 3: 35 32 44 28 13 50 20 19 7 26 31 10 21 37 23 48
Route 4: 1 3 9 34 14 16 46 42 47 15 36 4

Problem: **CON 534** Generation Optimum Reached: 9206 Solution: 602.82

Route 1: 17 21 26 46 23 29 11 40 37 13 35 44 8 20
Route 2: 43 48 10 2 25 9 28 15 18 39 7 16 30 34 47
Route 3: 49 45 12 14 4 38 50 41 33 24 42 3 27 32 31 5
Route 4: 36 6 1 19 22

Problem: **CON 535** Generation Optimum Reached: 8170 Solution: 575.57

Route 1: 43 29 11 24 14 30 8 21 7 36 50 39 48 33 5 46 9
Route 2: 35 18 44 20 47 10 32 31 12 13 2 41 23 15
Route 3: 42 22 3 17 1 49 26 25 19 37 6 38 45 16
Route 4: 40 34 27 28 4

Problem: **CON 536** Generation Optimum Reached: 8938 Solution: 524.23

Route 1: 24 5 17 34 37 50 26 13 10 3 43 39 48 49 41 12 4 42 25 47
Route 2: 19 8 23 38 2 30 18 28 35 11 29
Route 3: 6 45 44 36 7 21 20 32 22 1 33 46 40 14 15 16
Route 4: 31 9 27

Problem: **CON 537** Generation Optimum Reached: 3328 Solution: 596.92

Route 1: 46 15 44 10 17 31 38 18 12 21 50 25 5 48 6 28 27
Route 2: 7 23 30 41 24 19 49 32 40 16 13 2
Route 3: 14 9 1 29 47 8 3 22 35 11 43 39 45 4 33 42
Route 4: 26 20 37 36 34

Problem: **CON 538** Generation Optimum Reached: 3173 Solution: 546.17

Route 1: 18 25 27 42 36 37 24 4 9 2 43 50 29 39
Route 2: 35 31 6 12 41 22 21 14
Route 3: 3 8 47 46 20 49 48 15 26 19 17 33 34 7 11
Route 4: 40 23 38 28 44 32 13 1 30 10 5 16 45

Problem: **CON 539** Generation Optimum Reached: 7223 Solution: 599.97

Route 1: 7 3 4 34 16 27 45 47 8 36 31 5 28 25 18 30 12 10
Route 2: 29 13 22 23 37 21 20 17 14 1 40 26 24 50
Route 3: 46 33 6 43 9 42 19 38 41 15 44 49 39 32
Route 4: 35 11 48 2

Problem: **CON 580** Generation Optimum Reached: 2156 Solution: 907.97

Route 1: 20 47 6 29 40 16 37
Route 2: 28 13 10 49 26 46
Route 3: 46 50 11 3 27 43 42

Route 4: 7 19 2 31
Route 5: 24 14 45 36 41
Route 6: 18 23 39 8 1 21 22
Route 7: 35 15 48 32
Route 8: 33 12 9
Route 9: 17 34 38 4
Route 10: 30 5 44 25

Problem: **CON 581** Generation Optimum Reached: 6296 Solution: 808.34

Route 1: 47 21 28 18 35 50 23 12
Route 2: 40 11 5 41 44 22
Route 3: 17 39 24 13 38 27
Route 4: 14 33 37 6
Route 5: 48 36 34 31 29 25
Route 6: 10
Route 7: 1 42 32 15 8 20
Route 8: 26 46 4 19
Route 9: 2 16 30
Route 10: 3 9 43 49 7 45

Problem: **CON 582** Generation Optimum Reached: 6935 Solution: 758.96

Route 1: 26 40 5 11 9
Route 2: 50 14 18 20
Route 3: 13 44 32 4 16 49
Route 4: 39 45 10 29
Route 5: 7 12 36 2 28 37 22
Route 6: 23
Route 7: 15 31 3 43 24 38 27 25
Route 8: 34 47 41 33 30
Route 9: 21 46 42 8 6 19
Route 10: 48 17 35 1

Problem: **CON 583** Generation Optimum Reached: 21766 Solution: 908.73

Route 1: 43 5 29 41 22 30
Route 2: 42 47 15 12 24
Route 3: 25 39 49 8 6
Route 4: 4
Route 5: 27 33 36
Route 6: 17
Route 7: 38 45 18 11 40
Route 8: 2
Route 9: 35 32 48

Route 10: 28 50 20 19 7 26 31 10 37
Route 11: 44 13 21 23
Route 12: 1 3 16 46 14 34 9

Problem: **CON 584** Generation Optimum Reached: 10466 Solution: 812.13

Route 1: 14 4 38 50 41 33
Route 2: 1 6 12 28 15 18 39 7 16
Route 3: 17 26 46 5
Route 4: 47 30 34 21
Route 5: 43 10 2 25 9
Route 6: 48 45 49
Route 7: 20 29 37 40 11 23
Route 8: 8 13 35 44
Route 9: 36 3 42 24 27 19 22
Route 10: 31 32

Problem: **CON 585** Generation Optimum Reached: 11156 Solution: 820.94

Route 1: 9 43 11 29 5 46 18
Route 2: 15 2 41 23 44
Route 3: 14 30 8 21 7
Route 4: 24 36 50 39 48 33
Route 5: 20 47 10 32 31 12 13
Route 6: 1 49 26 25 19 45
Route 7: 42 22 3 17 16
Route 8: 35 40 34 27
Route 9: 38 37 6 28 4

Problem: **CON 586** Generation Optimum Reached: 23847 Solution: 725.23

Route 1: 5 21 20 7 45 44 6
Route 2: 25 4 42 46 40
Route 3: 47 26 13 10 43 3 50 31
Route 4: 23 30 2 38 8
Route 5: 11 35 28 18
Route 6: 12 41 49 48 39 24
Route 7: 19 34 37 9 17
Route 8: 29 16 15 27
Route 9: 36 32 22 1 33 14

Problem: **CON 587** Generation Optimum Reached: 18906 Solution: 871.66

Route 1: 30 23 1 29 47 3 8 36
Route 2: 43 11 35 22 20 37
Route 3: 50 25 5 48 6

Route 4: 16 46 15 17 31 10 44 32
Route 5: 27 28 24 41 2
Route 6: 19 49 42 26
Route 7: 40 13 39
Route 8: 21 12 18 38
Route 9: 7 9 14 34
Route 10: 45 4 33

Problem: **CON 588** Generation Optimum Reached: 6294 Solution: 810.82

Route 1: 36 24 37 42 18
Route 2: 49 48 15 26 19 20
Route 3: 39 29 31 16 45 11
Route 4: 4 9 2 43 50
Route 5: 5 10 30 1 13 17 46
Route 6: 7 33 28 44 32 38
Route 7: 34 23 40 35
Route 8: 47 8 3
Route 9: 6 12 41 14
Route 10: 21 25 27 22

Problem: **CON 589** Generation Optimum Reached: 7257 Solution: 860.70

Route 1: 29 48 5 28 31 36 11
Route 2: 10 4 8 47 45 27 16 7
Route 3: 22 12 20 17 14 1
Route 4: 44 15 21 37
Route 5: 25 18 30
Route 6: 19 42 38 41
Route 7: 24 26 40 9 43 39
Route 8: 46 32 49 23 13
Route 9: 3 34 35 2
Route 10: 50 33 6

APPENDIX B

BEST ROUTES FOR SALHI AND NAGY (1999) INSTANCES

Problem: **CMT1X** Generation Optimum Reached: 1997 Solution: 484.22

Route 1: 11 38 9 50 16 2 29 21 34 30 39 10 49 5
Route 2: 47 4 18 13 41 40 19 42 44 45 33 15 37 17 12 46
Route 3: 32 1 22 20 35 36 3 28 31 26 8 48 23 7 43 24 25 14 6
Route 4: 27

Problem: **CMT1Y** Generation Optimum Reached: 1009 Solution: 607.996

Route 1: 11 38 5 49 9 50 16 29 21 34 30 10 39 33 45 15 44 42 37 17 4
Route 2: 47 18 41 19 40 13 14 6 48 23 24 43 7 26 8 32
Route 3: 12
Route 4: 2
Route 5: 25
Route 6: 27 1 22 31 28 3 36 35 20
Route 7: 46

Problem: **CMT2X** Generation Optimum Reached: 8673 Solution: 757.90

Route 1: 38 65 66 11 59 8
Route 2: 17 40 72 58 10 31 25 55 18 50 9 39 12
Route 3: 16 63 1 43 42 64 41 56 23 49 24 32 44 3 51
Route 4: 75 30 48 5 29 45 27 52 46 34
Route 5: 26 67 68 2 28 22 61 21 74 4
Route 6: 7 35 53 14 19 54 13 57 15 37 20 70 60 71 69 36 47
Route 7: 6 33 73 62

Problem: **CMT2Y** Generation Optimum Reached: 7805 Solution: 725.78

Route 1: 7 53 11 66 65 38 31 10 58 35 8 46
Route 2: 17 51 6 49 24 44 32 50 18 55 25 9 39 72 12 26
Route 3: 75 68 2 22 64 42 43 41 56 23 63 16 3 40
Route 4: 67 34 52 27 13 57 15 37 20 70 60 71 69 36 47 74 30
Route 5: 33 1 73 62 28 61 21 48 5 29 45 4
Route 6: 14 59 19 54

Problem: **CMT3X** Generation Optimum Reached: 4144 Solution: 774.74

Route 1: 50 33 81 51 9 35 71 65 66 20 32 90 63 64 49 36 46 47 19 11 62 10 52
Route 2: 26 12 76 77 3 79 78 34 29 24 68 80 54 4 55 25 39 67 23 56 75 72 73 21 40
Route 3: 53 28 27 69 1 30 70 31 88 7 48 82 8 45 17 84 5 99 59 96 6 89
Route 4: 94 95 97 92 37 98 100 91 16 86 38 44 14 42 43 15 57 41 22 74 2 58
Route 5: 13 87 93 85 61 60 83 18

Problem: **CMT3Y** Generation Optimum Reached: 10032 Solution: 823.97

Route 1: 76 77 3 79 81 51 9 71 35 34 78 33 50 1 69 4 39 23 67 25 55 24 29 68 80 28

Route 2: 89 6 96 99 59 92 37 98 85 91 100 14 43 15 41 22 74 75 56 72 73 21 54 12 26 40 2 57 42 97
Route 3: 53 58 13 87 95 93 44 38 86 16 61 84 17 45 8 46 47 36 49 64 11 62 88 52
Route 4: 27 31 70 30 20 65 66 32 90 63 10 19 48 82 7 18 83 60 5 94

Problem: **CMT4X** Generation Optimum Reached: 9459 Solution: 1103.59

Route 1: 112 53 40 149 80 68 116 77 3 79 121 24 55 25 67 39 139 130 109 12
Route 2: 146 132 51 120 9 103 66 20 128 131 32 90 63 126 108 10 62 107 11 64 49 143 36 47 124 46 52
Route 3: 6 94 117 95 92 37 93 147 89 106 123 19 48 114 8 125 45 17 113 84 5 118
Route 4: 105 26 138 28 69 1 122 30 70 101 31 127 88 148 7 82 83 60 59 13
Route 5: 58 137 87 98 100 91 85 99 104 96 97 42 57 145 41 22 133 23 56 75 74 73 72 110 4 54 150
Route 6: 76 50 102 33 81 135 35 136 65 71 34 78 129 29 134
Route 7: 111 27 18 61 16 86 141 44 119 140 38 14 142 43 15 144 2 115 21

Problem: **CMT4Y** Generation Optimum Reached: 4649 Solution: 1247.09

Route 1: 59 93 82 48 124 46 47 123 19 107 11 10 70 88 7 106 18 83 60 61 85 91 100 37 98 96 6
Route 2: 112
Route 3: 1 33 78 9 120 135 35 136 65 71 103 20 30 131 32 90 126 63 64 49 143 36 114 125 45 17 84 5
Route 4: 13 95 117 87 137 2 115 73 21 72 74 23 41 145 15 43 14 44 141 16 86 140 38 142 42 92 99 104
Route 5: 58
Route 6: 94
Route 7: 147 89 146 27 69 148 62 108 128 66 51 122 101 52 8 118 113 119 97 144 57 75 39 139 110 40
Route 8: 127 31 132 111 76 116 77 3 79 129 68 80 150 109 54 130 55 25 134 24 29 121 34 81 102 50
Route 9: 53 105 22 133 56 67 4 149 12 26
Route 10: 28
Route 11: 138

Problem: **CMT5X** Generation Optimum Reached: 6386 Solution: 1237.58

Route 1: 96 99 59 92 95 94 6 147 18 153 106 7 90 11 143 36 47 124 46 45 17 113 84 118 114 8 82
Route 2: 122 120 9 71 65 136 35 135 34 78 81 33 129 3 77
Route 3: 27 111 28 154 138 12 109 150 134 54 130 55 25 139 39 67 23 56 74 72 21 149
Route 4: 105 152 137 117 104 93 85 91 100 97 2 115 57 15 43 38 140 86 141
Route 5: 146 69 101 70 116 80 68 20 66 128 131 32 126 63 64 49 107 19 123 62 159 10 148 88 52
Route 6: 89 151 37 98 5 60 83 48 125 61 16 44 119 14 142 42 144 87 13

Route 7: 26 24 29 121 76 50 157 51 103 79 158 110 155 4 75 133 22 41 145 73 40 58 53
Route 8: 132 1 102 30 108 31 127 112 156

Problem: **CMT11X** Generation Optimum Reached: 13269 Solution: 1152.05

Route 1: 67 69 70 71 73 76 68 77 74 72 75 78 80 79 40 43 45 51 50 48 42 39 38 41 32 35 36 34 33 30 28 21 94
Route 2: 99 59 65 61 62 64 66 63 60 55 54 57 47 49 46 44 37 29 31 27 24 22 25 19 23 26 20 17 16 12 91
Route 3: 84 117 83 113 90 114 18 118 108 109 97 115 110 98 56 58 53 52 8 13 14 15 11 10 9 7 6 5 4 3 2 81
Route 4: 120
Route 5: 1
Route 6: 107 104 103 116 100 96 93 92 89 85 112
Route 7: 119 88
Route 8: 87 86 111 82
Route 9: 95 102 101 106 105

Problem: **CMT11Y** Generation Optimum Reached: 16214 Solution: 925.87

Route 1: 70 71 73 74 75 72 78 77 79 39 45 51 50 47 46 44 29 32 35 36 34 31 30 33 27 24 22 25 28 23 26 21 20 19 16 17
Route 2: 106 104 69 67 76 68 80 53 54 57 59 65 61 62 64 66 63 60 56 58 55 52 40 43 48 49 41 42 38 37 110
Route 3: 88 82 86 92 97 109 115 98 116 96 84 117 113 83 2 1 3 4 5 10 11 15 14 13 12 8 9 7 6 108 118 112 81
Route 4: 120 105 107 103 100 99 101 102 93 94 114 18 90 91 89 85 111
Route 5: 87 95
Route 6: 107 104 103 116 100 96 93 92 89 85 112
Route 7: 119

Problem: **CMT12X** Generation Optimum Reached: 4167 Solution: 802.56

Route 1: 43 42 41 40 44 45 48 51 50 46 47 49 52 31 35 37 38 39 36 33 32 30 28 26
Route 2: 7 3 5 75 1 2 4 6 9 8 10 11 17 23 21 22 25 27 29 34 24 20
Route 3: 90 91 89 88 85 84 83 82 81 78 76 71 70 73 77 79 80 72
Route 4: 87 86 63 65 67 62 74 61 64 55 54 53 56 58 60 59 68 69
Route 5: 98 96 95 94 92 93 97 100 99 12 14 16 15 19 18 13
Route 6: 66 57

Problem: **CMT12Y** Generation Optimum Reached: 14661 Solution: 664.42

Route 1: 21 23 26 28 30 34 32 33 36 39 38 37 35 31 29 27 25 24 22 20
Route 2: 91 87 86 83 82 84 85 96 95 94 92 93 97 100 99 98 88 89 90
Route 3: 49 43 41 40 59 60 58 56 53 54 55 57 42 44 45 46 48 51 50 52 47
Route 4: 63 66 69 68 64 61 72 80 79 77 73 70 71 76 78 81 74 62 65 67

Route 5: 75 1 2 4 3 5 7 6 8 9 12 14 16 15 19 18 17 13 11 10